VOLATILITY SMILES, SURFACES AND OPTION PRICES

Introduction

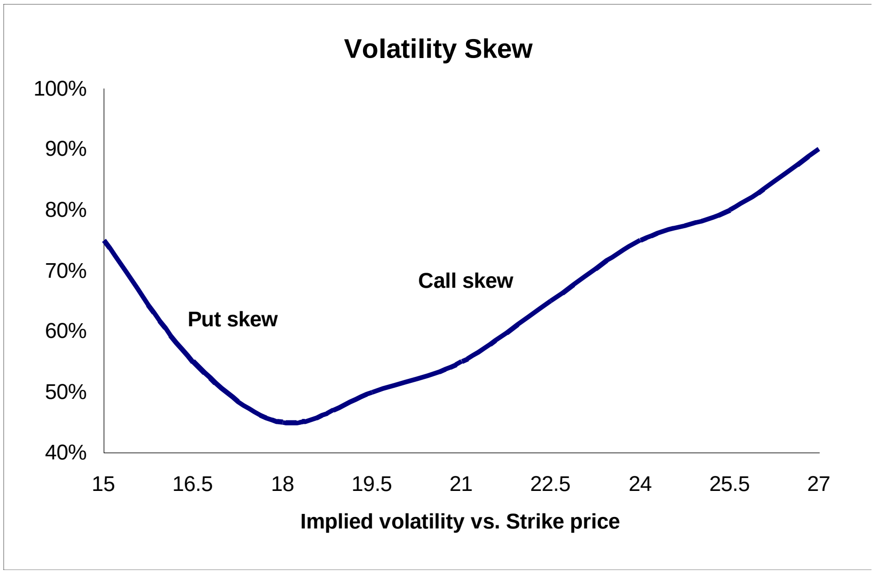
After nearly 30 years from its publication, the Black Scholes options pricing model is still the most commonly used amongst market practitioners. Some of the main appeals of the BS model are their simplicity, robustness and ease of implementation. The only unknown parameter in the Black-Scholes model is the volatility of the underlying. Therefore, if we have a market price for a particular option, we can extract the implied volatility according to the Black-Scholes model.

Even though it is widely recognized that the standard Black-Scholes model suffers from serious imperfections, the Black Scholes model is so popular in energy and financial markets that it is standard for practitioners to quote option prices in terms of actual market prices or BS implied volatilities. Traders are fully aware of the limitations, but rather than replacing the model, they have been able to adequately "tweak" it in order to account for certain imperfections and inaccuracies, particularly in the way that the model deals with volatilities.

Market practitioners often quote option prices, and therefore express their market views, in terms of the implied volatility for a given strike and maturity. The Black-Scholes model assumes that implied volatility is constant and homogeneous for options on the same underlying with different strikes and maturities. However, in practice the implied volatility of call or put options at a given date is a function of the strike price and exercise dates.

In the case of energy markets, the implied volatility of options on different forward contracts usually decays as the time to maturity increases. The range of implied volatilities for options with different times implied volatilities for a given strike, usually the at-the-money strike, is known as the term structure of volatilities.

The structure of volatilities for different strikes for a given maturity tends to have the shape of a ``smile'' or a "skew". The volatility skew is usually built with ATM and OTM puts and calls, which are the most liquid options traded in the market. The chart below depicts a hypothetical volatility skew. We can observe two different "slopes" on either side of the at-the-money strike of 18.The *put skew* affects options with strikes less than 18 and the *call skew* greater than 18. Skews can be positive, negative, or zero.



When implied volatilities are higher for OTM strikes for puts and calls as they go further out-of-the-money, the shape resembles a "smile", and therefore the term "volatility smile". Regardless of the actual shape of the structure of implied volatilities vs. the strike, we will use the term "smile" even though other authors refer to them as "smirks", "frowns" or just "skews".

Implied Volatility Surface

If we combine the volatility smiles for multiple maturities, we can create a table or matrix whose elements represent volatilities for different strikes and maturities. The collection of implied volatilities for different strikes and maturities is called the "implied volatility surface" because if we plot that table, we can see a "surface" of volatility smiles.

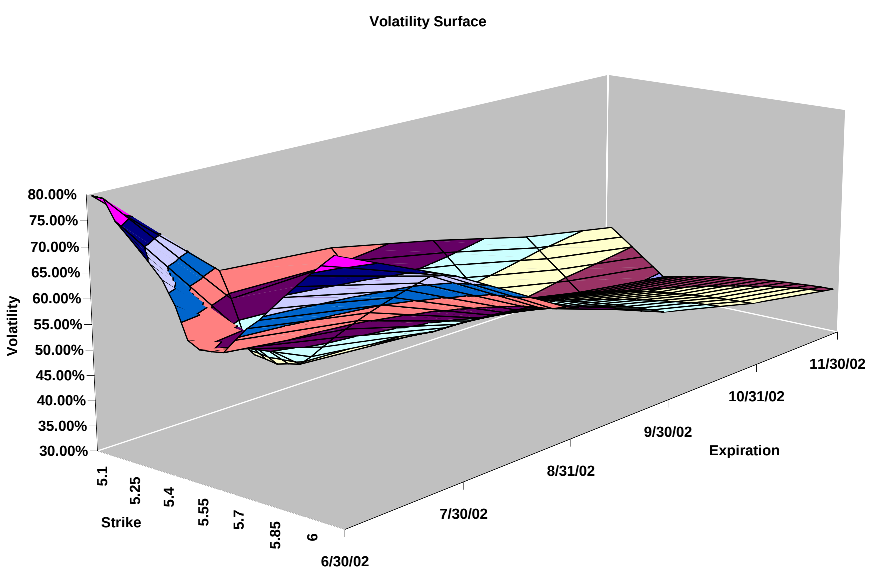


Figure 1 reproduces the nature of a typical volatility surface. On the vertical axis we can see the implied volatility for a given strike and maturity combination.

As we mentioned before, volatility surfaces are characterized by being non-flat. The volatility surface just represents a “snapshot” at any point in time of the option prices quoted for different strikes and maturities. As the market changes, the volatility surface also changes its shape as a response to changes in option prices.

Implied Volatility Matrix and Option Prices

The Black-Scholes model assumes that volatility is constant across strikes and maturity dates. However, in the world of energy options, this is a very unrealistic assumption. Option prices for different maturities change drastically, and option prices for different strikes also experience significant variations.

The Black-Scholes model does not provide for the ability to match a set of option prices at any given moment. In order to be able to quote (and price) option prices for varying levels of strikes and maturities, practitioners use different single volatilities depending on the strike and maturity of the option. Therefore, if we are pricing a book of options with different strikes and maturities, using a unique implied volatility for each underlying could lead to serious pricing errors.

Financial engineers have attempted to answer the question about how to modify the underlying pricing model in order to take this phenomenon into account. The shortcoming of BS have led to a considerable amount of research and alternative models that attempt to describe the dynamics of the underlying asset in terms of alternative distributions which match a representation of the implied volatility surface. However, these models are not currently used by many practitioners due to a series of limitations regarding their use in the trading floor. These limitations include a large computational burden, added complexity, and lack of the market information needed to calibrate the models.

Therefore, many market participants in the energy derivatives markets still use Black-Scholes or slight deviations from it, and use a volatility surface to price a set of European calls and puts for a range of strikes and maturities. It is important to point out, that we are just using a different volatility to price a particular option, and we are not changing our assumptions about the underlying process.
In a way, using implied volatility smiles allows traders to use the wrong parameter in the wrong formula in order to obtain the correct market price.

Implied volatilities are still the standard way of quoting option prices, and the starting point to price different options contracts because they are expressed in a way that practitioners find familiar (the opposite is true for more complex models that treat volatility as a stochastic factor) , and they are directly observable without making any model assumptions about how they vary by strike and time to maturity.

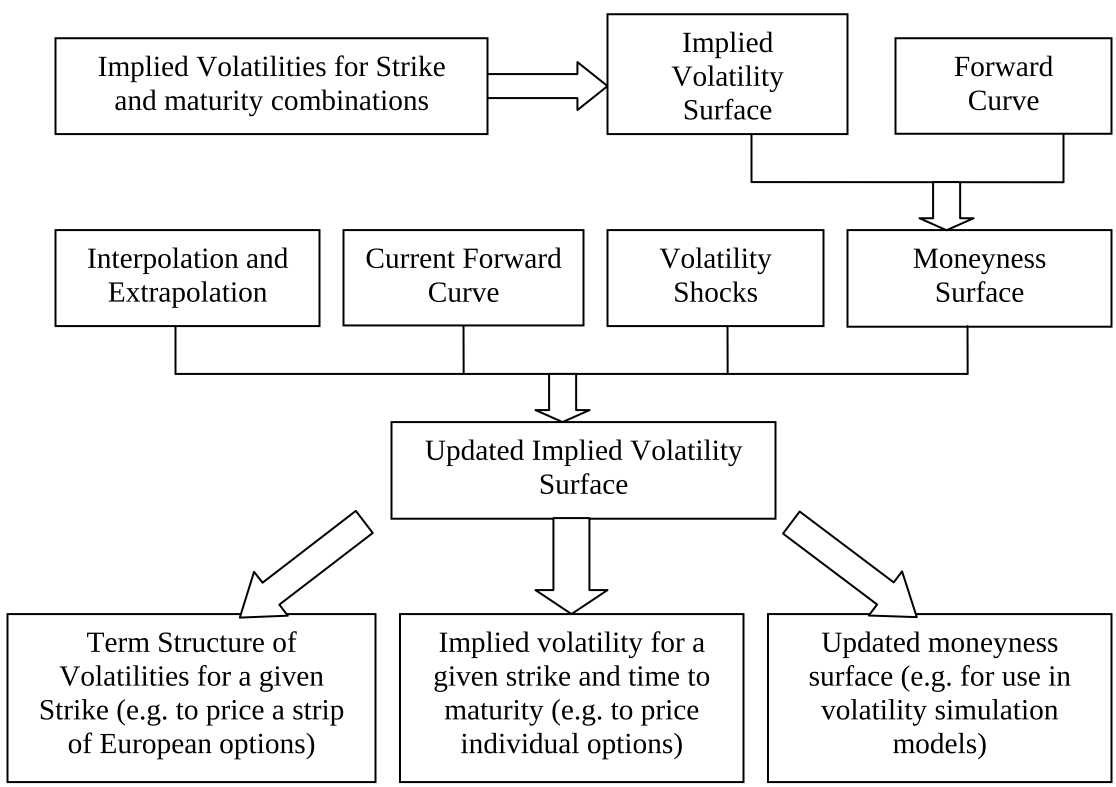
Building a Implied Volatility Smile Surface through Calibration

So far we have assumed that the volatility surface can be built from observed option quotes. However, there is a caveat. Most options markets do not have liquid quotes for many OTM options. In practice, it is common that the only observed volatilities in the market are coming from a limited set of broker quotes which are quite restrictive in terms of coverage of strike prices and forward contracts. For longer term maturities and deep OTM options, the data tends to be quite scattered and sometimes unreliable unless it is updated regularly and there are actual traded contracts at those prices.

Therefore, in order to build and update the surface, we need to resort to creative methods to use all the available information in the market in terms of quoted options and prevailing forward prices.

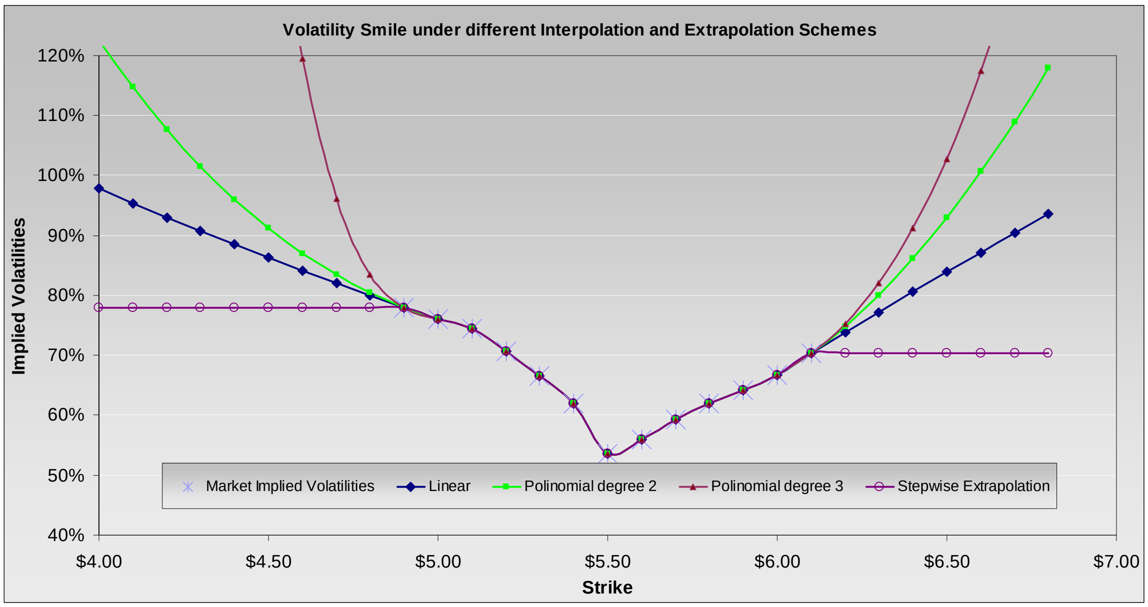
Many firms operating in options markets build a volatility surface with end-of-day closing prices to perform mark-to-market calculations. An options book may have options with different strikes than the ones currently traded. For example, the ATM options at some point in time may become well ITM or OTM in the future. When the risk management or back-office personnel have to mark-to-market open options positions for strikes that become very ITM or OTM as forward prices change, implied volatilities need to be obtained for those strikes.

A similar problem is faced by option market makers and option traders. An options market-maker needs to be able to provide bid and offer prices on the options that he is making a market for. At any point in time, he needs to have an updated implied volatility surface to price options with different strikes and maturities. In order to be able to use that surface in a trading environment, the surface needs to be adjusted as a response of changes in market forward prices and a limited set of implied volatilities for the more liquid options.



Deterministic Volatility Surface

As we mentioned before, traders often treat volatility as a function of the strike and maturity of the contract by building a volatility surface. In order to build the volatility surface to be used to price a continuous range of options based on those limited option quotes, we need to interpolate between different strikes, and extrapolate for strikes beyond the largest one available for each maturity, and below the lowest one for each maturity.



Updating Volatility Smiles and Surfaces for changes in market prices

After the volatility surface has been calibrated, we need to be able to "update" it in order to take into account changes in market forward prices and implied volatilities for certain traded contracts.

Therefore, instead of working with volatilies as a function of strikes (for a fixed contract expiry) internally we convert those into volatilities as a function of a moneyness parameter, given a specified forward curve.

Once that implied volatility surface has been updated, we may be interested in extracting implied volatilities as a function of either strikes, "moneyness" or expiration dates.

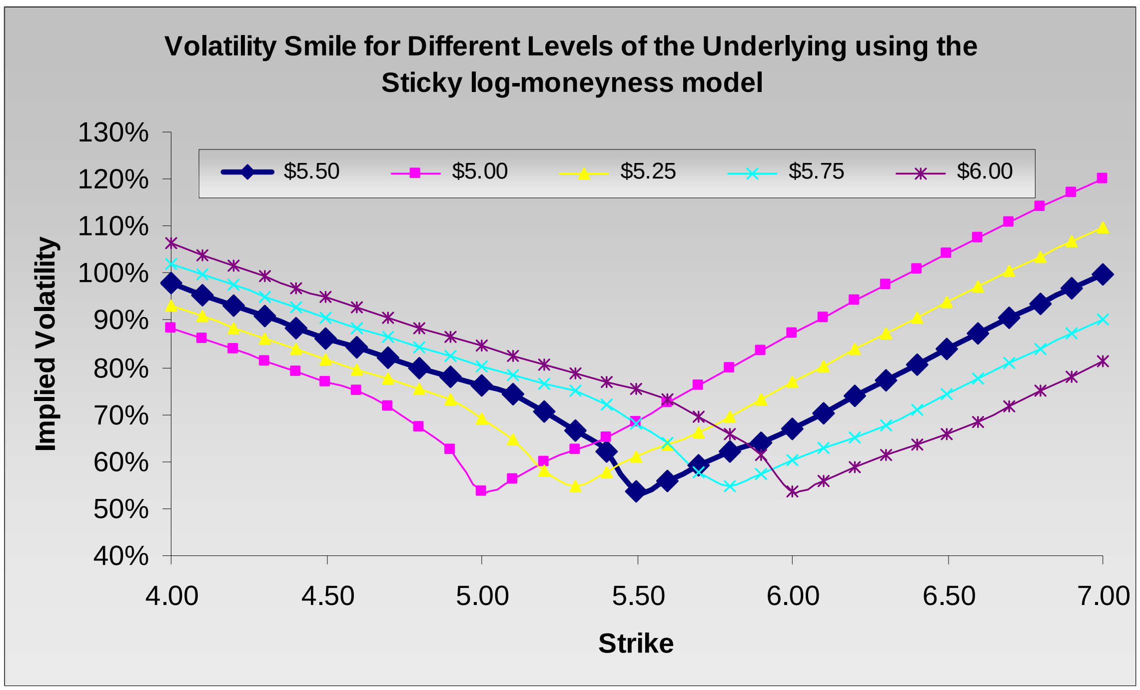
In order to update the volatility surface, we need to define what constitutes a reasonable model to describe the evolution of implied volatilities for a particular strike as market forward prices change. There are three main approaches used by practitioners:

a. **Sticky Moneyness (or log-Moneyness)**

The basic assumption of this approach is that, while the implied volatility as a function of strike does not adequately capture volatility market movements, the implied volatility as a function of "moneyness" parameter does. For example, if yesterday an option with strike K was in the money today it might be out of the money due to a movement in the market forward curve. Hence yesterday's associated implied volatility is not today's correct implied volatility. In essence, due to a move in the market forward curve we should move along the smile.

Rather than absolute "moneyness", many traders express moneyness based on the log of the forward price vs. the strike. For a strike K and a forward price F_T with expiry T we define the associated moneyness as $\log(F_T/K)$.

The implied volatility for a particular combination of strike price and maturity is calculated from the "moneyness" surface by translating that combination of strike and current market prices into "moneyness" terms and using a particular set of interpolation or extrapolation schemes.



In this figure we can see how the smile adjusts to changes in the level of the underlying. If we were valuing options with the strikes defined in the x axis, we would be using different volatilities depending on the original smile calibrated when the underlying was trading at $5.5 and the current level of the underlying. If we fixed the strike in relation to the current forward prices at any point in time, the surface remains constant for options with the same log-moneyness when the forward curve changes.

b. Sticky Delta

Another common approach is to define the degree of moneyness in terms of the delta of the options. This implies that the volatility is stuck to the delta of a particular strike.

This approach has the advantage that we can take into account the passage of time when building the moneyness surface, as time enters the calculation of the delta. However, this comes with an added pitfall because deltas depend on the volatility parameter, and therefore we need to assume a starting level of volatility to determine the delta that would give the right implied volatility in the surface.

c. Sticky Strike

Another approach assume that the implied volatility for an option with a given strike and maturity does not react to changes in forward prices. This approach is used in equity markets under certain conditions, but it is not very applicable to the energy world. This would be equivalent of not using the prevailing forward curve to update the volatility surface.

Updating Volatility Smiles and Surfaces for changes in market volatilities for liquid options

Volatility surfaces need to be updated when forward prices change, or the implied volatility levels experience variations. Changes in the shape of the implied volatilities for different points in the surface are usually highly correlated across strikes and maturities due to various arbitrage relationships.

Therefore, if we have updated information on the implied volatilities for the most liquid options traded in the market, we can treat those as a "volatility shock" and shift the other point in each smile or the whole surface by a particular amount based on the original shock.

Implied Probability Distributions and Volatility Smiles.

The existence of a "smile" usually implies that the probability distribution of the underlying has fatter tails and is more peaked around the mean than the lognormal distribution, which means that both large and small moves in the underlying are more likely than what the lognormal distribution predicts.

If we have an OTM call with a strike Y, the option will expire in-the-money when the underlying price is above Y. If the implied volatility is higher for OTM options, that means that the probability of exercise must be higher than the one implied by the lognormal distribution assuming at-the-money strikes.

If we have an OTM put with strike X, the option will expire in-the-money when the underlying price is below X. That means that if the implied volatility is higher than for ATM, the implied probability of the price falling below X is also higher.

In equities, it is common to see a "volatility skew", in which OTM puts have much higher volatilities than OTM calls. There are two explanations. After the crash of 1987, investors' perception of market crashes changed considerably, and since then, they have been assigning much higher probability to market "crashes" than to market "booms". The second explanation is that as the stock price of a particular company declines, the leverage ratio (the ratio of debt to equity) increases and therefore the risk of investing in the company increases considerably.

For many commodities where the price is supported by non-market effects (e.g. government intervention or production costs), it is common to see a flat or negative skew for puts and a pronounced positive skew for calls, due to the risk of price spikes.

The shape of the volatility smile can also provide information about the correlation of price movements and changes in the implied volatility. For certain commodities, it is reasonable to expect that price increases are more likely to be associated with higher implied volatilities than price decreases. For example, in the oil markets, when price goes up it is usually due to increased geopolitical instability, and this is commonly associated with higher levels of volatility. In the case of Natural Gas and Electricity markets, large price increases are usually the result of supply or demand shocks that tend to be followed by higher implied volatilities for traded options.

Implied Volatilities for Put and Calls with the same strike and maturity

In order to price a European call or put option with the same strike and maturity with the BS model, the same volatility should be used This is always true for European options when Put-Call parity holds, and it does not depend on the future probability distribution of the underlying due to the fact that it is based on simple arbitrage arguments. It is also approximately true for American options. Therefore, when we are talking about implied volatilities, it does not matter whether we are referring to calls or puts, because they should be the same.