Forward Models

- One Factor

$$\frac{dF(t\ \ T)}{F(t\ \ T)} \qquad (t)\,dw(t)$$

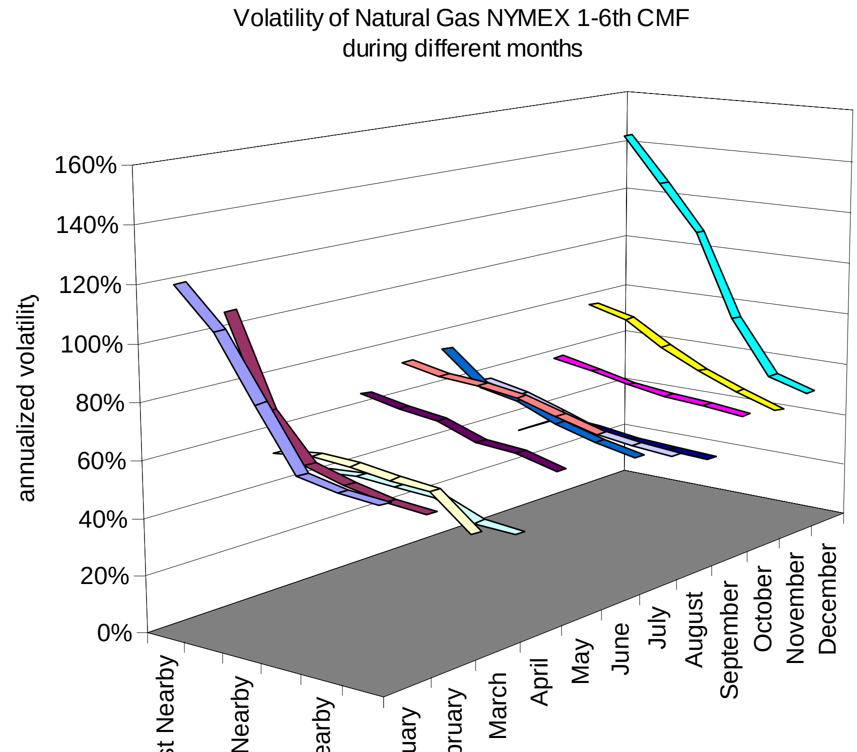
- Multi-Factor

$$\frac{(\qquad)}{\qquad)} \qquad ($$

$\sigma_T(t)$ volatility of forward (price)

$\sigma(t) \equiv \sigma_t(t)$ forward volatility

Seasonal Volatility at a Glance



Constant Maturity Futures Volatility Grid

Observation Month ↓	Constant Maturity Month → nearby 1	nearby 2	nearby 3	nearby 4	nearby 5	nearby 6	nearby 7	nearby 8	nearby 9	nearby 10	nearby 11	nearby 12
Jan	97.8%	88.0%	69.6%	51.9%	49.6%	48.6%	47.4%	36.7%	36.0%	34.3%	32.9%	32.0%
Feb	85.5%	60.9%	48.0%	44.6%	42.7%	41.5%	33.0%	31.7%	29.3%	27.4%	26.2%	25.5%
March	44.6%	47.0%	45.8%	44.4%	43.5%	33.6%	32.7%	30.5%	28.6%	27.8%	26.6%	13.8%
April	35.1%	35.1%	34.2%	33.7%	29.2%	28.7%	27.2%	25.4%	24.8%	24.1%	15.8%	13.9%
May	54.1%	53.0%	50.7%	46.8%	45.9%	42.9%	40.3%	39.2%	38.5%	35.8%	34.7%	31.9%
June	61.6%	60.4%	59.6%	58.1%	54.3%	50.9%	49.9%	50.1%	53.5%	52.3%	51.0%	50.0%
July	63.4%	52.1%	50.2%	44.9%	41.2%	39.5%	39.4%	36.0%	36.4%	36.7%	37.2%	37.0%
Aug	49.6%	47.7%	43.0%	39.0%	37.2%	36.0%	31.7%	29.8%	28.7%	28.5%	27.6%	27.3%
Sept	28.6%	34.9%	34.2%	31.9%	31.3%	30.7%	29.0%	27.9%	27.1%	26.2%	25.8%	23.4%
Oct	52.6%	48.4%	45.7%	44.2%	43.3%	42.0%	39.7%	38.7%	37.5%	37.4%	27.5%	27.2%
Nov	61.7%	59.8%	52.5%	47.0%	41.7%	38.1%	37.6%	37.3%	36.9%	28.1%	27.8%	26.2%
Dec	116.6%	104.1%	88.8%	61.6%	44.2%	41.5%	40.4%	39.3%	30.2%	28.9%	27.1%	25.5%
Averages	67.0%	60.0%	53.9%	47.1%	42.9%	40.0%	38.0%	36.1%	33.8%	31.8%	30.2%	28.5%

Seasonal Multi-Factor Model

- Seasonality Assumptions (e.g. monthly granularity)

volatility $$\sigma_T(t) \Rightarrow \sigma_i(\alpha)$$

i : number of relevant expiry months (e.g. *N* total).
α : different observation periods (e.g. 12 total).

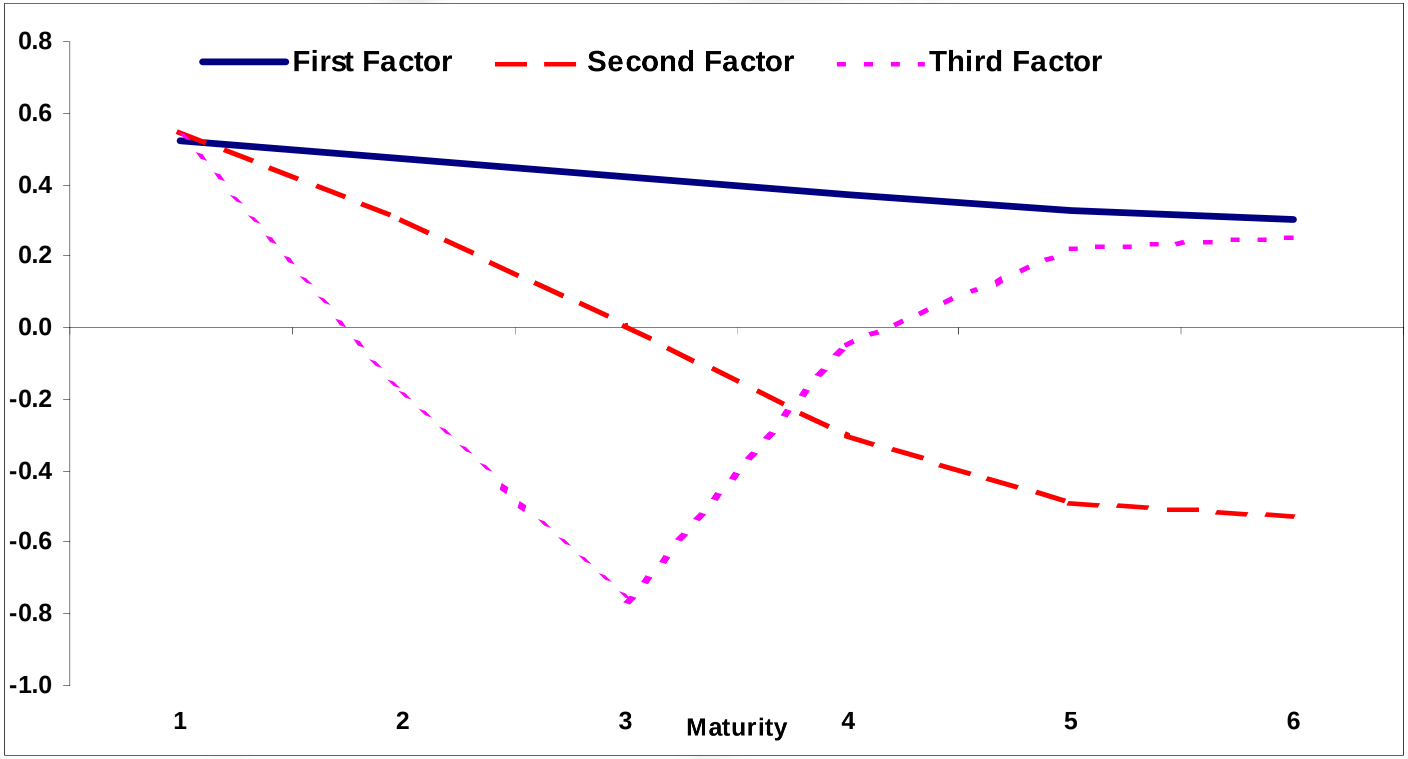
correlation $$\rho_{ij}(\alpha) \equiv \frac{\mathrm{cov}_{ij}(\alpha)}{\sigma_i(\alpha)\sigma_j(\alpha)}$$

Multi-Factor Model

- Principal Components Analysis (PCA)
  - diagonalize the covariance matrix
    => eigenvalues: $\{\lambda_i : i = 1,\ldots, N\}$
  - identify the few independent factors which account for a significant percentage of the total variance
    => principal components: $\{d\hat{w}_i(t) : i = 1,\ldots, n << N\}$
  - express the original stochastic drivers in terms of the principal components, e.g.

=> $$\frac{dF(t,T_i)}{F(t,T_i)} \cong \sum_j^n c_{ij}(t)\lambda_j d\hat{w}_j(t)$$

Sample (PCA) Factor Loadings



Joint Simulation of Spot and Forward Prices

- Two-factor mean reverting model:
  - A prompt-month forward + Another independent stochastic driver
  - Mean-reverting to the prompt-month forward.

$$d\log\{S(t)/F_1\} = -a\log\{S(t)/F_1\}dt + \sigma_s(t)\left\{\rho\, dw_1(t) + \sqrt{1-\rho^2}\, dw_s(t)\right\}$$

  - a - spot mean-reversion rate
  - Spot-to-prompt-month correlation
  - A set of forward volatilities of the spot price $\sigma_s(t)$
  - A simulated prompt-month forward price

Selection of Price Process

1. Different energy markets exhibit different price behavior and therefore we will need to select the correct model to accurately describe a given market.
2. The right models of price processes provide greater accuracy in option pricing and in the calculation of hedge statistics.

If the price process does not fit, you must use it with extreme caution

Basis Risk

- **Modeling the Spread as a Lognormal Asset Price**
- **Modeling the Spread as a Normal Variable (Wilcox)**
- **Two-factor Approach: Spread as a Joint LogNormal (Garman)**
- **Correlation as a measure of co-movement in energy markets**
- **One-factor vs. two factor approaches**

#1 Modeling the Spread as a Lognormal Asset Price

- Spread is model as an asset price, lognormally distributed
- Advantages
  - Usual Black-Scholes type formulas apply.
  - Option value is only sensitive to changes in the spread levels, not the underlying prices
  - Sensitivity to the Spread explicitly captured
- Limitation
  - Assumes that the probability that the spread will ever become negative is nil.
  - Spread fluctuation sizes would increase for large spreads and decrease for small ones (volatility is expressed as a % of price)
  - Assumes that spread delta is applied to buying and selling equivalent futures positions of each side of the spread
  - Does not account for Mean Reversion of the Spread

Spread as Lognormal

- A. Calculate the volatility of the spread directly like if it was an individual asset price
  - Problem: Can not deal with negative prices

- B. Approximate volatility of the spread through individual components

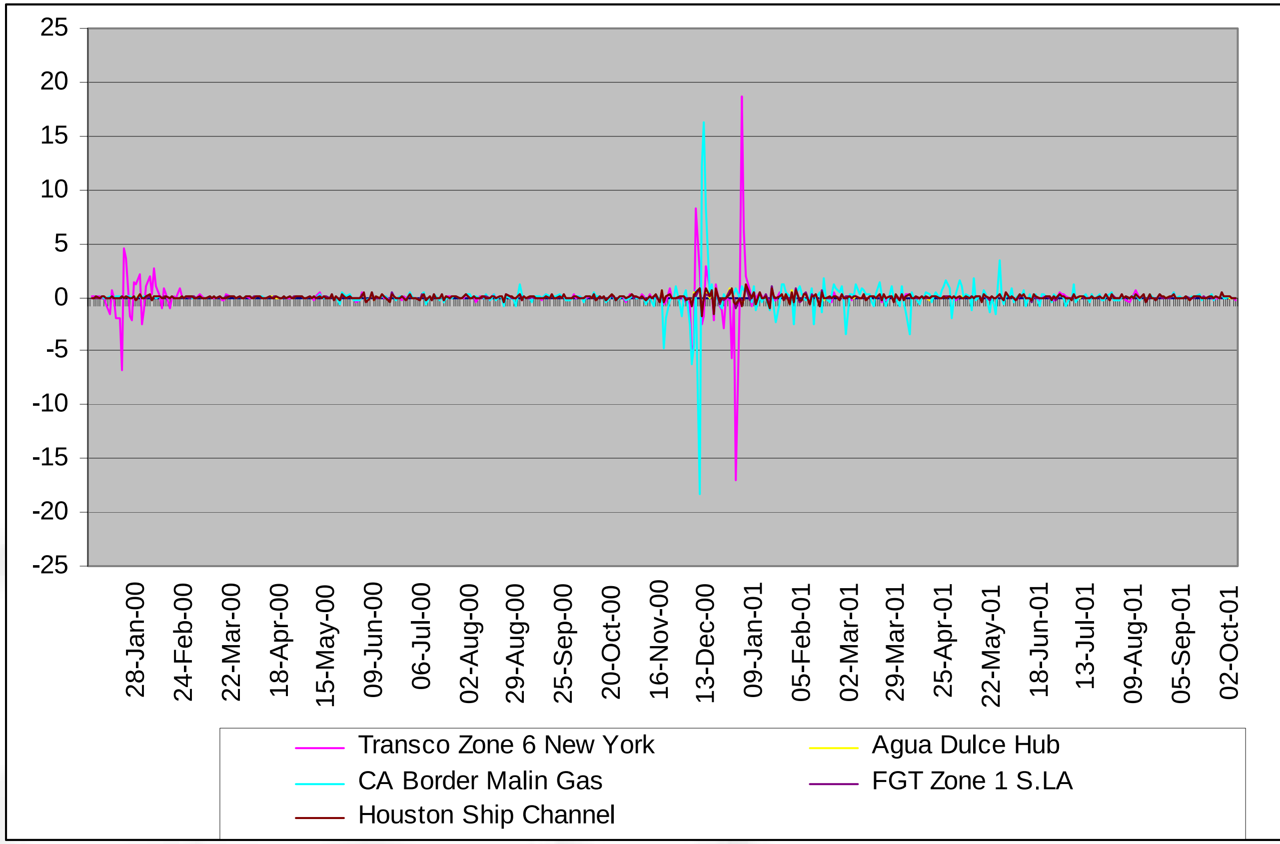
$$\sigma_S = \sqrt{\sigma_{f_1}^2 + \sigma_{f_{12}}^2 + 2\rho\,\sigma_{f_1}\,\sigma_{f_2}}$$

- The volatility of the spread is a combination of the volatility of the individual components and the correlation between them
- Assumes that the difference of two correlated lognormals is also lognormal. Unfortunately this is not true.

#2 Modeling the Spread as a Normal Variable (Wilcox)

- Assumes spread is a normally distributed variable
- Advantages:
  - Spreads can be positive or negative
  - Spread volatility is measured in $, not as a %
- Since the payoff depends only on the value of the spread at expiration, there is no effect from a move in the market that leaves the spread level unchanged.
- Limitation:
  - Only one delta to the spread. (If the spread comprised a high-volatility asset with a price of five and a low-volatility asset with price one (and therefore a spread of four), it is apparent that the appropriate hedge must seriously differentiate between the two assets, and no single delta would suffice.
  - Does not account for Mean Reversion of the Spread

Modeling Spread as a Normal Variable



Basis Statistics vs. NYMEX (1/4/00 - 5/10/01)	Transco Zone 6 New York	Agua Dulce Hub	CA Border Malin Gas	FGT Zone 1 S.LA	Houston Ship Channel
Average	$ 0.0010	$ 0.0003	$ 0.0011	$ 0.0001	$ 0.0002
Standard Deviation	$ 1.49	$ 0.23	$ 1.52	$ 0.23	$ 0.25
Annual St. Dev.	$ 23.68	$ 3.69	$ 24.16	$ 3.60	$ 3.97
Min	$ (16.95)	$ (1.46)	$ (18.38)	$ (1.32)	$ (1.74)
Max	$ 18.65	$ 1.12	$ 16.33	$ 1.01	$ 1.14
Skewness	1.21	-0.41	0.00	-0.19	-0.90
Kurtosis	93.11	6.97	85.94	6.17	[illegible]

#3 Two-factor Approach: Spread as a Joint LogNormal (Garman)

- The prices of the two assets evolve according to a stochastic process where the two prices are lognormally distributed and correlated.
- Each spot price is a random walk governed by a lognormal diffusion process (Black-Scholes framework)
- Interest rates, volatilities and correlations are deterministic
- The *correlation* is specified with a number between –1 and +1, exclusive.

Differential Process

- The differential N-factor process the dynamics of the N spot prices is:

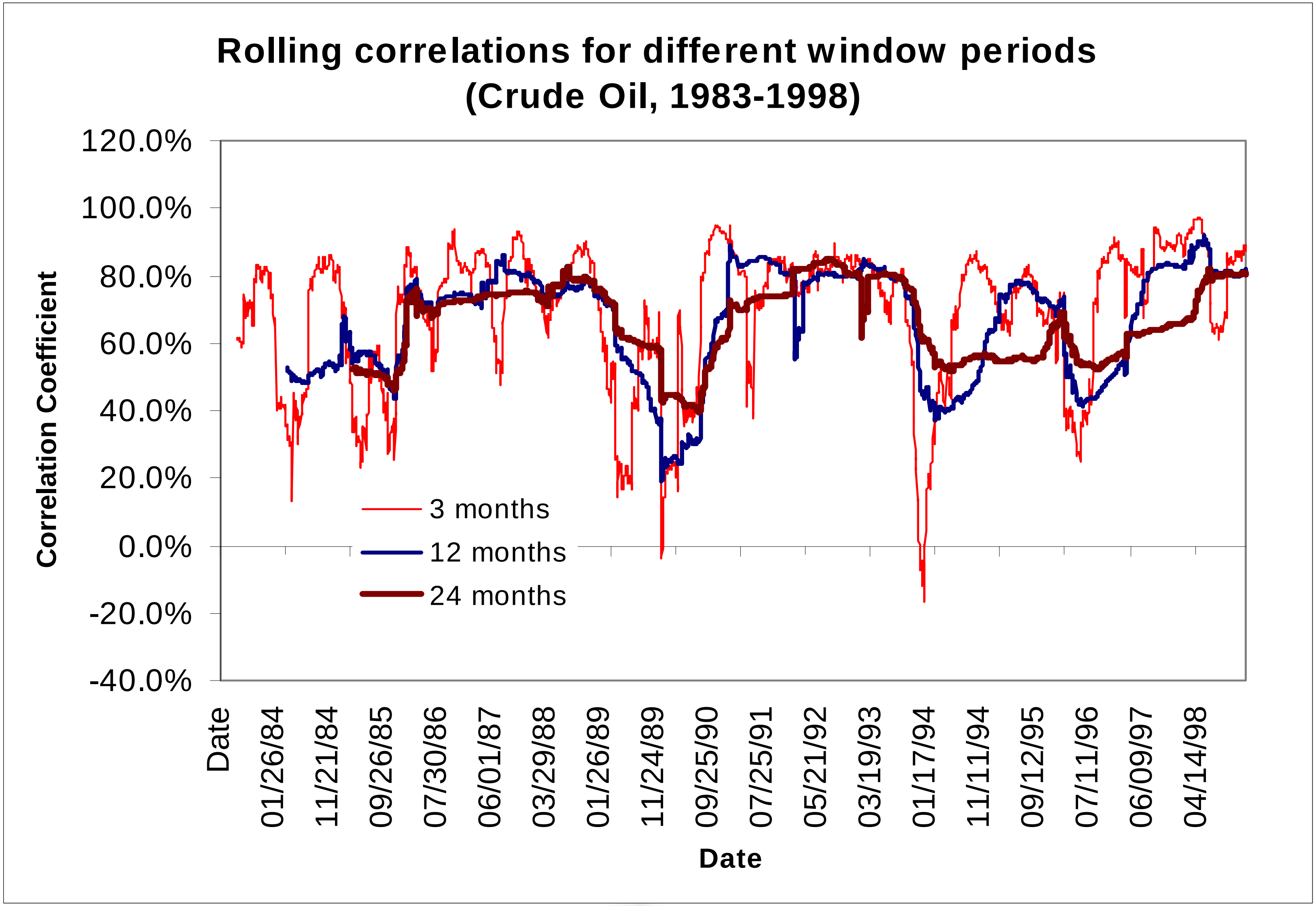
$$dS_i = \mu_i S_i \, d\tau + \sigma_i S_i \, dZ_i \;, i = 1, N$$

- Where μ_i *and* σ_i are the drift rate and volatility of the ith factor.

- The dynamics of different $S_i(\tau)'s$ are correlated because $\Delta z_i = \varepsilon_i \sqrt{\Delta \tau}$,with the ε_i drawn from a jointly normal correlated distribution

Is correlation a good measure of co-movement?

- Correlation is a rather simplistic measure of co-movement between two time series.
- Correlation between energy prices is dependent on:
  - Time – Depending on the time period that we are trying to analyze (e.g. summer vs. winter), the correlation will be different
  - Maturity – Depending on the maturity of the future contracts, or spot/future prices, the correlation will be different
  - Nature of the price shock – Depending on the nature of the price shock in one of the variables, the movement in the other series may be quite different (e.g. electricity and gas correlation under price spikes vs. normal market conditions)


Rolling correlations for different window periods
(Crude Oil, 1983-1998)
120.0%
100.0%
80.0%
60.0%
40.0%
20.0%
0.0%
-20.0%
-40.0%
Correlation Coefficient
3 months
12 months
24 months
Date
01/26/84
11/21/84
09/26/85
07/30/86
06/01/87
03/29/88
01/26/89
11/24/89
09/25/90
07/25/91
05/21/92
03/19/93
01/17/94
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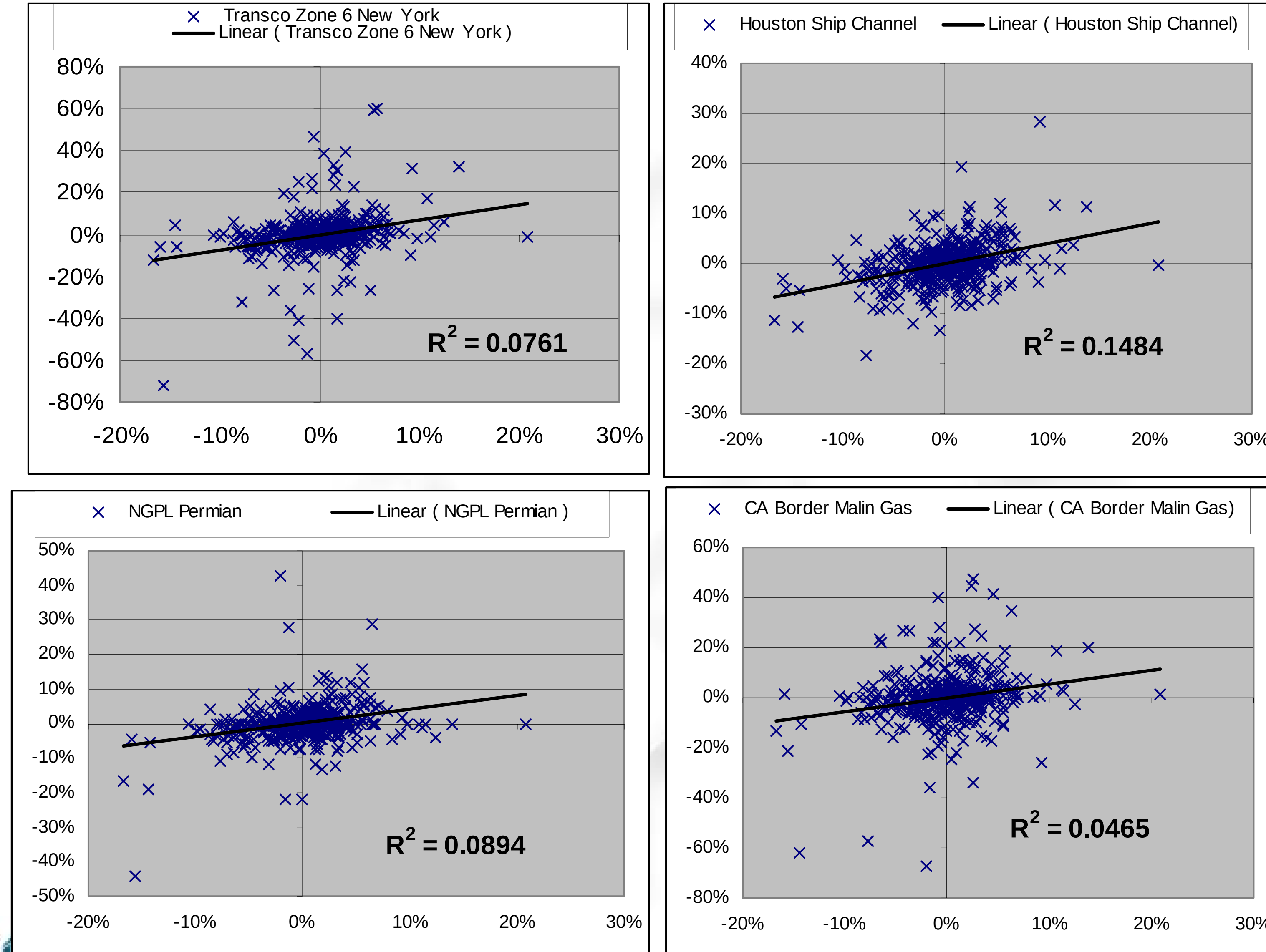
09/12/95
07/11/96
06/09/97
04/14/98
Date

Daily Returns between NYMEX and Different Locations



Data provided by Fame Energy

One-factor vs. Two-factor Approaches

- **Advantages of the Two-Price Models:**
  - Provides two deltas/gammas/vegas
  - Considers correlation and its risk
  - Can incorporate different mean-reversion rates for each price in the spread
  - Can be solved by 2-dimensional binomial tress or Numerical Integration
- **Limitations of Two-Price Models**
  - Sensitivity to the spread and the volatility of the spread not explicitly captured
  - Hedge ratios break down for extreme volatilities due to lognormality assumption

Taxonomy of Energy Derivatives Structures

Decomposing Contract Components

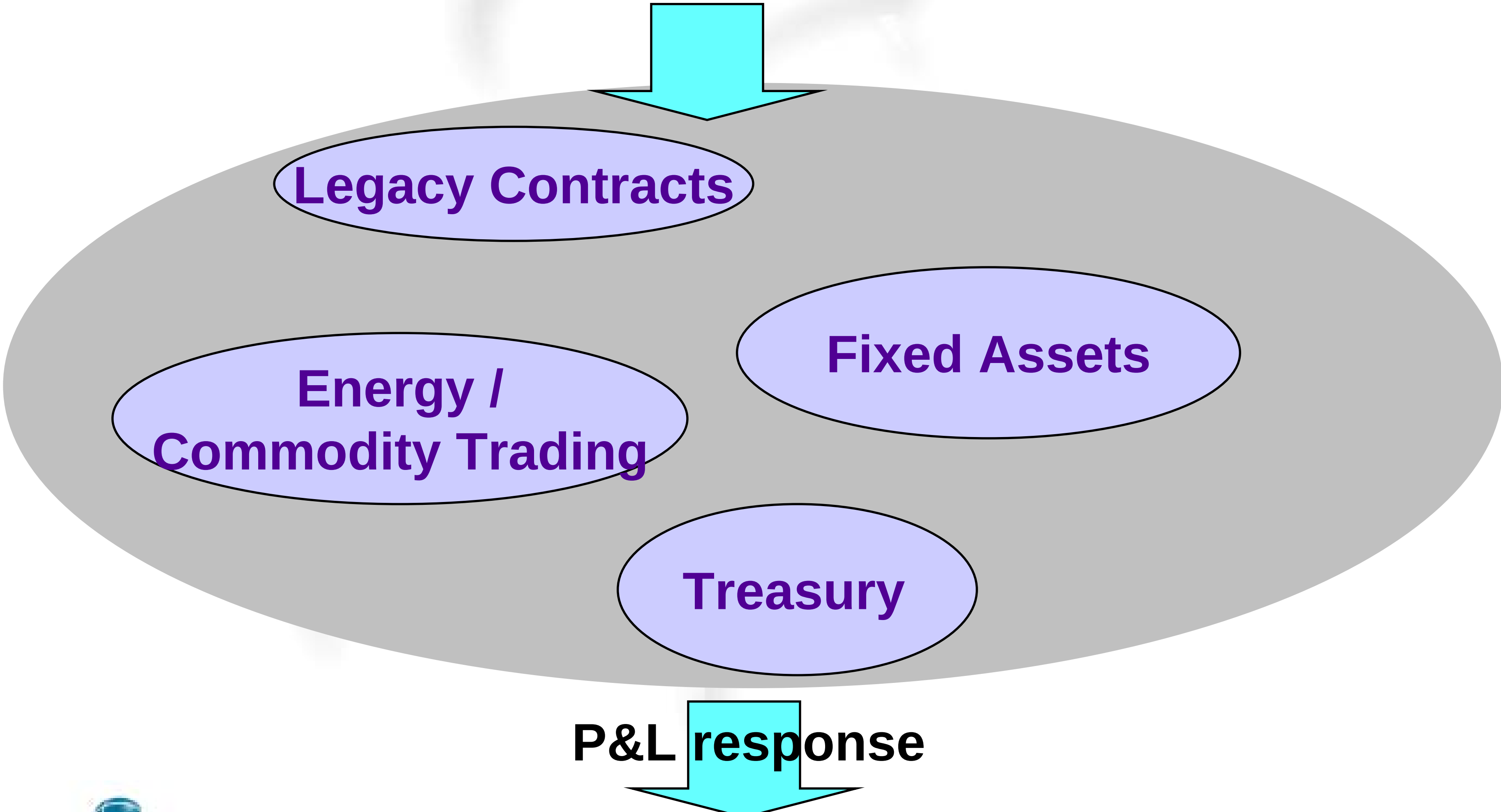
Derivative building block	+	Long or Short	+	Option Type	+	Strike Price Type	+	Exercise Style	+	Underlying Type	+	Payoff type

Building Block	Are we long or short	Option Type	Strike Price	Exercise Style	Underlying	Payoff type
Forward	Long	Call	Fixed	European	Spot	Simple (plain-vanilla)
Swap	Short	Put	Average of underlying	American	Forward	Digital
Option		Straddle	Index at future date	Bermudan	Swap	Barrier
Strip of options				Swing (co-dependent)	Spread	Average
					Option	

Fixed Assets as Derivatives

The Enterprise as a Derivative

State Variables: power prices, fuel prices, load, unit up/down



Generation Assets

- A gas or coal-fired generator may be viewed as a strip of spread options, where the spread is between the price of a MWH produced and the gas or coal required to produce it, suitably converted by an efficiency factor (e.g. tons of coal to BTUs to MWH produced), and the strike price of the options is the variable operating cost of the generator
- Extensions:
  - Generation assets with option to switch between two fuels
  - Generation with two possible delivery locations

Fixed Asset: An Oil Well

A oil well (coal mine, gas well) can be viewed as a strip of options to extract oil (I.e. oil “cashflows”) with strike price equal to the variable production cost and amount equal to the maximum production rate. If reserves are limited, then the well (mine) may be viewed instead as a swing option with the right to swing up to the maximal production from zero, with a total maximum “take” equal to the reserves.

Fixed Asset Valuation

Asset / Contract Clause	Derivative Structure
Thermal Units (Natural gas, Oil, Coal)	Spark Spread Options or Best-Of options
Nuclear and Baseload Units	Forward Contracts or Call Options
Transmission/Interconnectors	Locational Spread Options
Interruptible Clauses	Call and Put Options, Swing Options

Introducing asset constraints

- Start up/Shutdown Costs & Time
- Minimum Uptime/ Downtime
- Maximum Continuous Runtime
- Ramp-up/Ramp-down Rates
- Outages
  - Expected outage frequency
  - Average Duration
  - Standard deviation