

Modelling Energy Derivatives

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Presentation Outline

- The Unique Nature of Energy Price Behavior
- Energy Price Processes
- Forward Curve Models
- Modeling Basis Risk
- Taxonomy of Energy Derivatives Structures
- Modeling Fixed Assets as Derivatives
- Pricing Methodologies
- Lessons from Enron

Warning

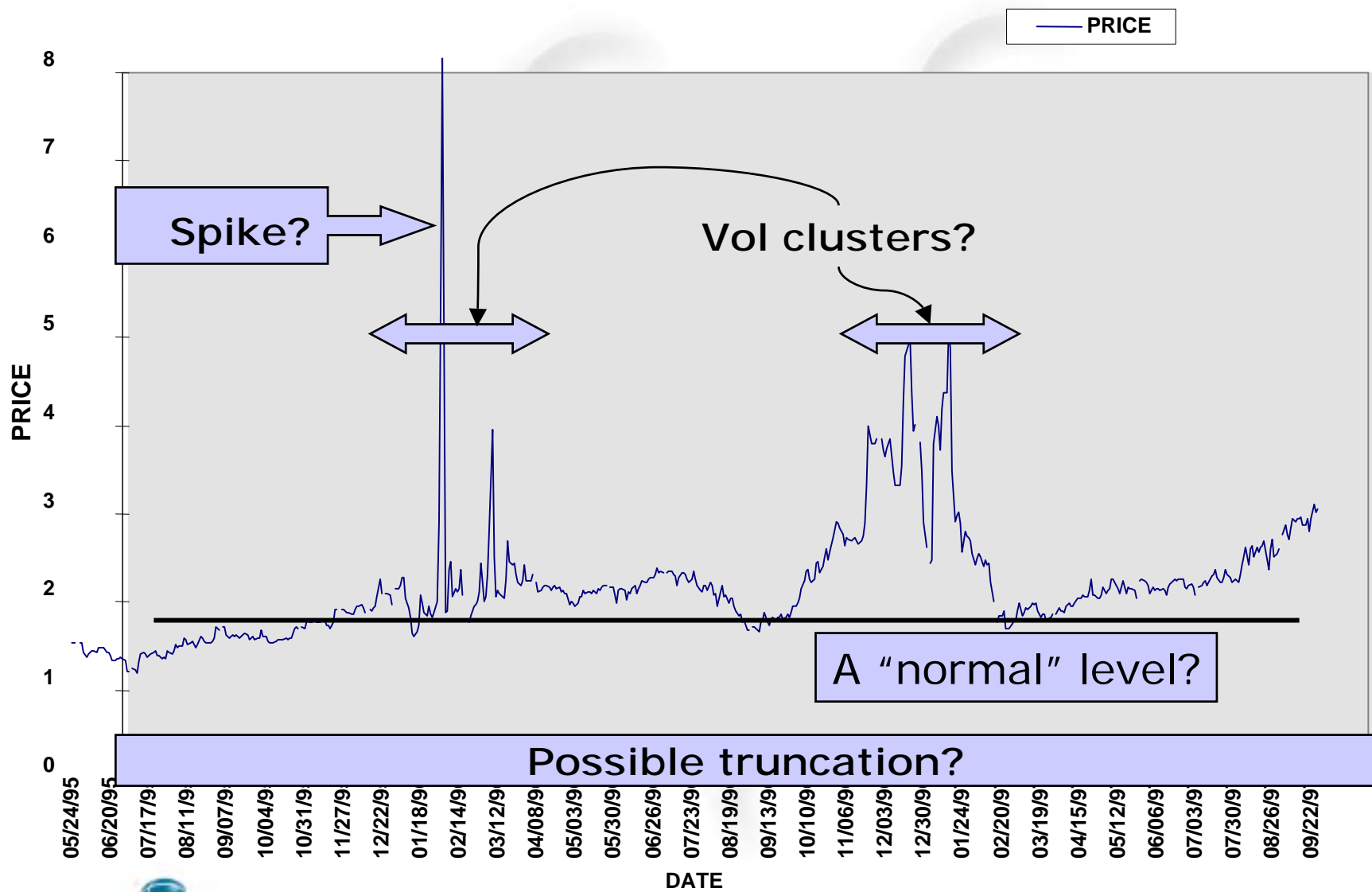
- Pricing, measuring, and managing the risk of energy contracts is an extremely complex task that should not be taken lightly



The Unique Nature of Energy Price Behavior

- Mean Reversion
- Truncated Distribution
- Price Spikes
- Fat tails
- Seasonality in both Prices and Volatility
- Time to maturity effect

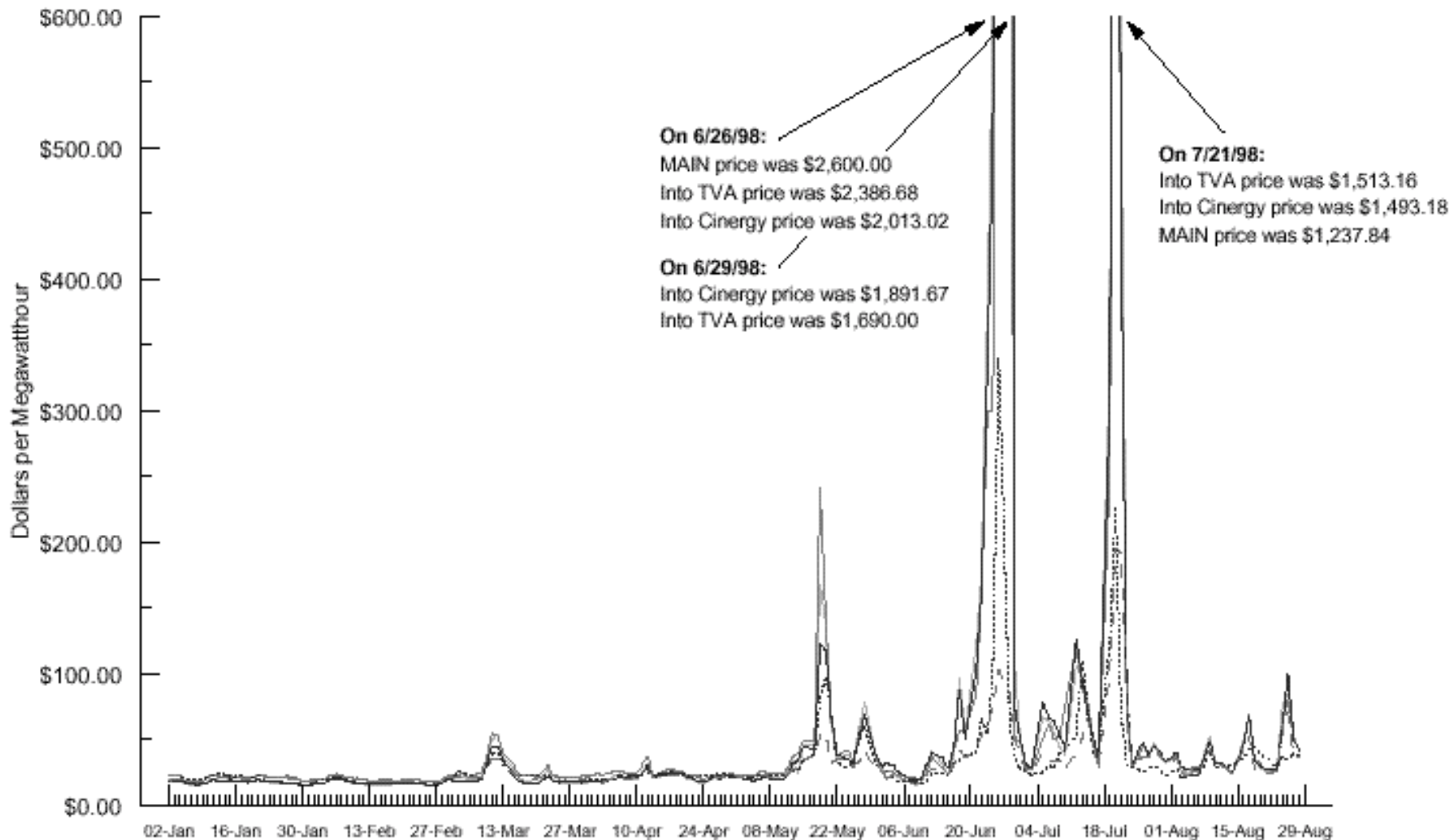
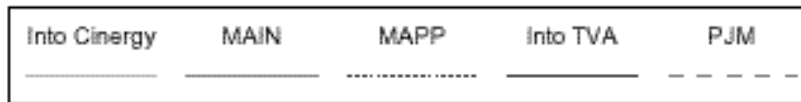
PERMIAN PRICE



Chicago, March 2002



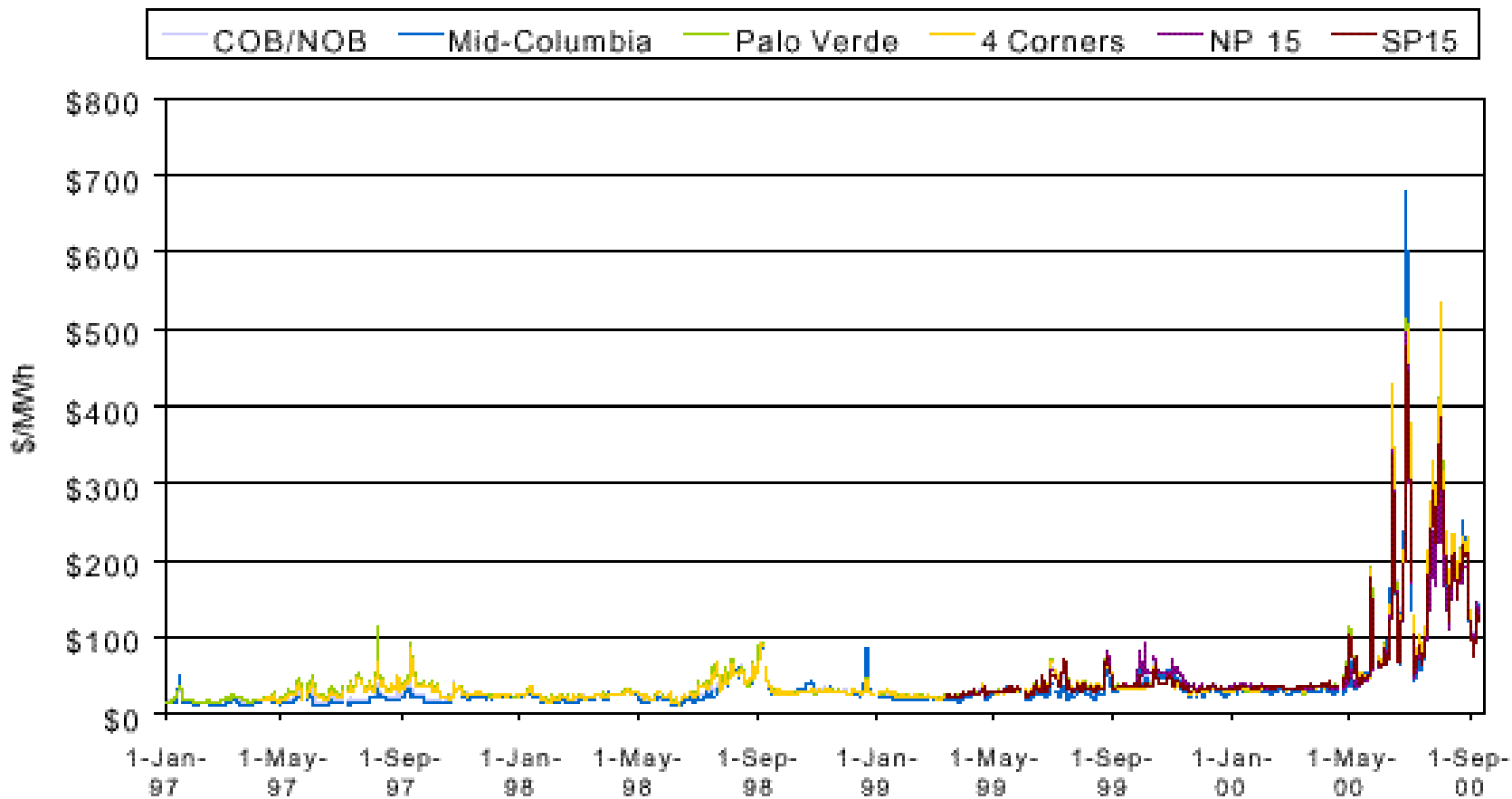
Prices for Next Day On-Peak Spot Electricity Markets (1998)



Source: *Power Markets Week*. Data through August 29, 1998.

Note: Prices are index prices for daily prescheduled, on-peak (16 hours) electricity.

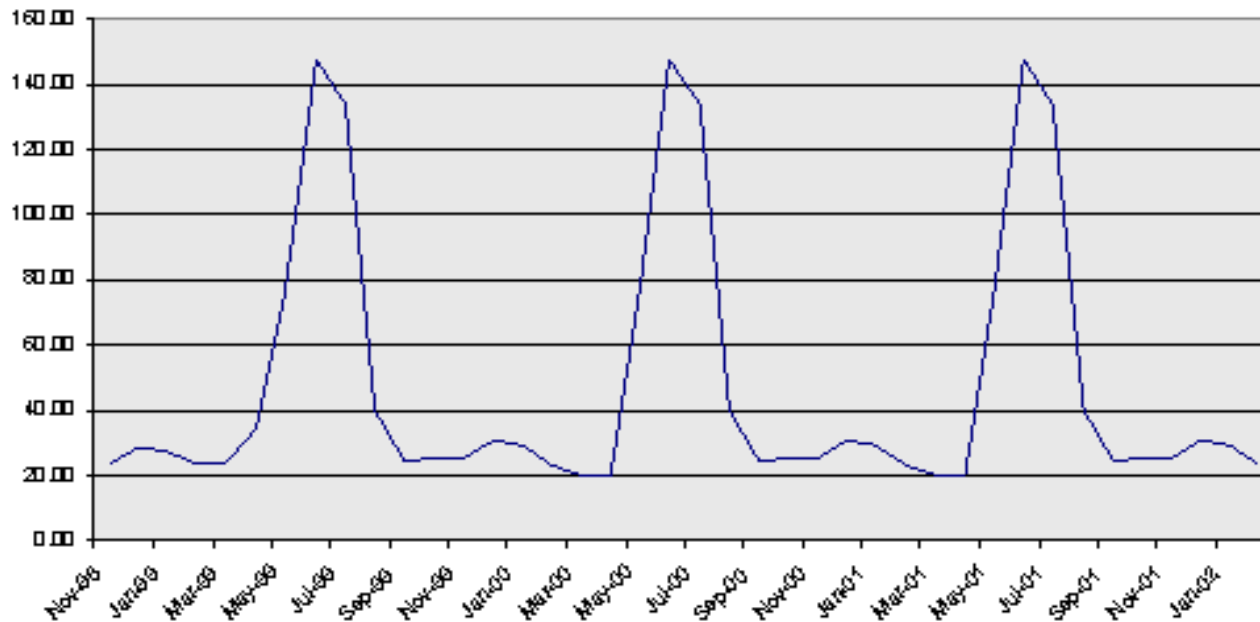
Daily Price Indices for Western Market Hubs, 1997-2000



Seasonality in both Prices and Volatility

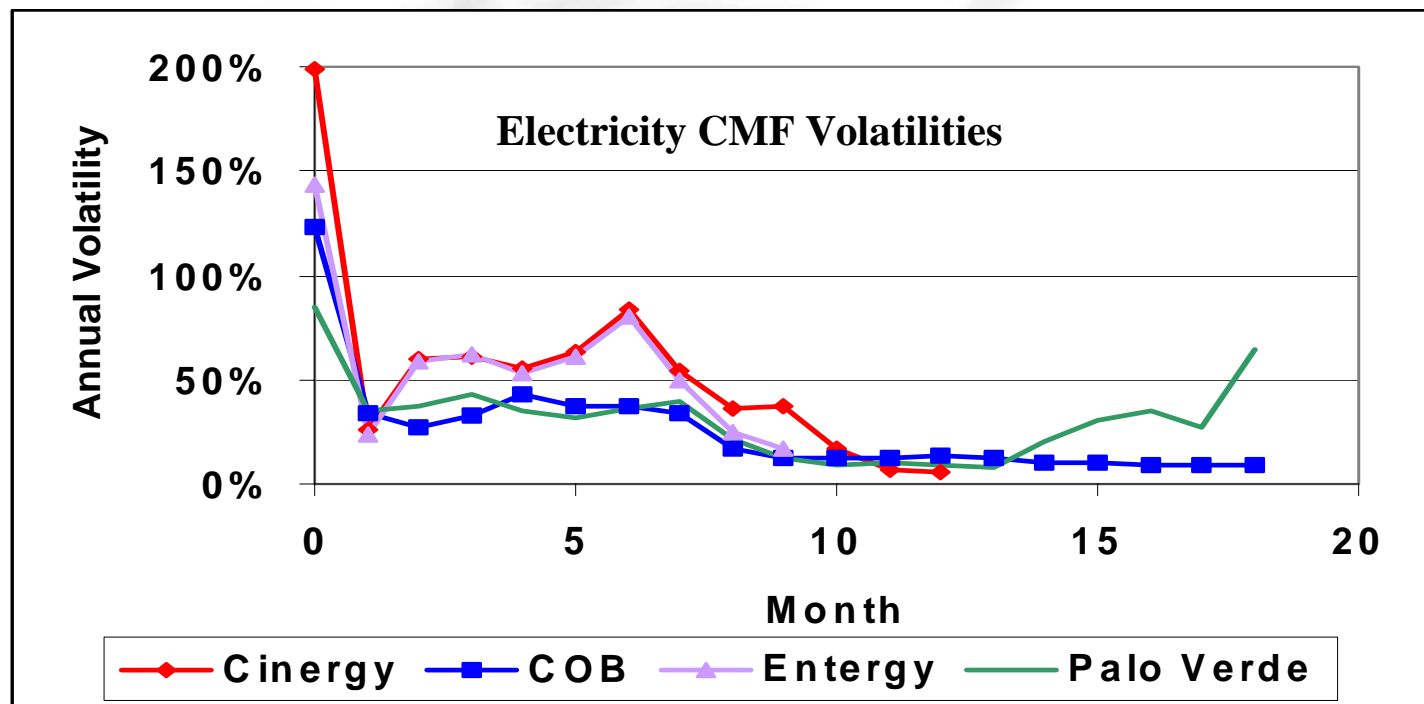
- In response to cyclical fluctuations in supply and demand, energy prices tend to exhibit strong seasonal patterns in both price and volatility.

Forward Price Curve for Electricity (November, 23rd, 1998)

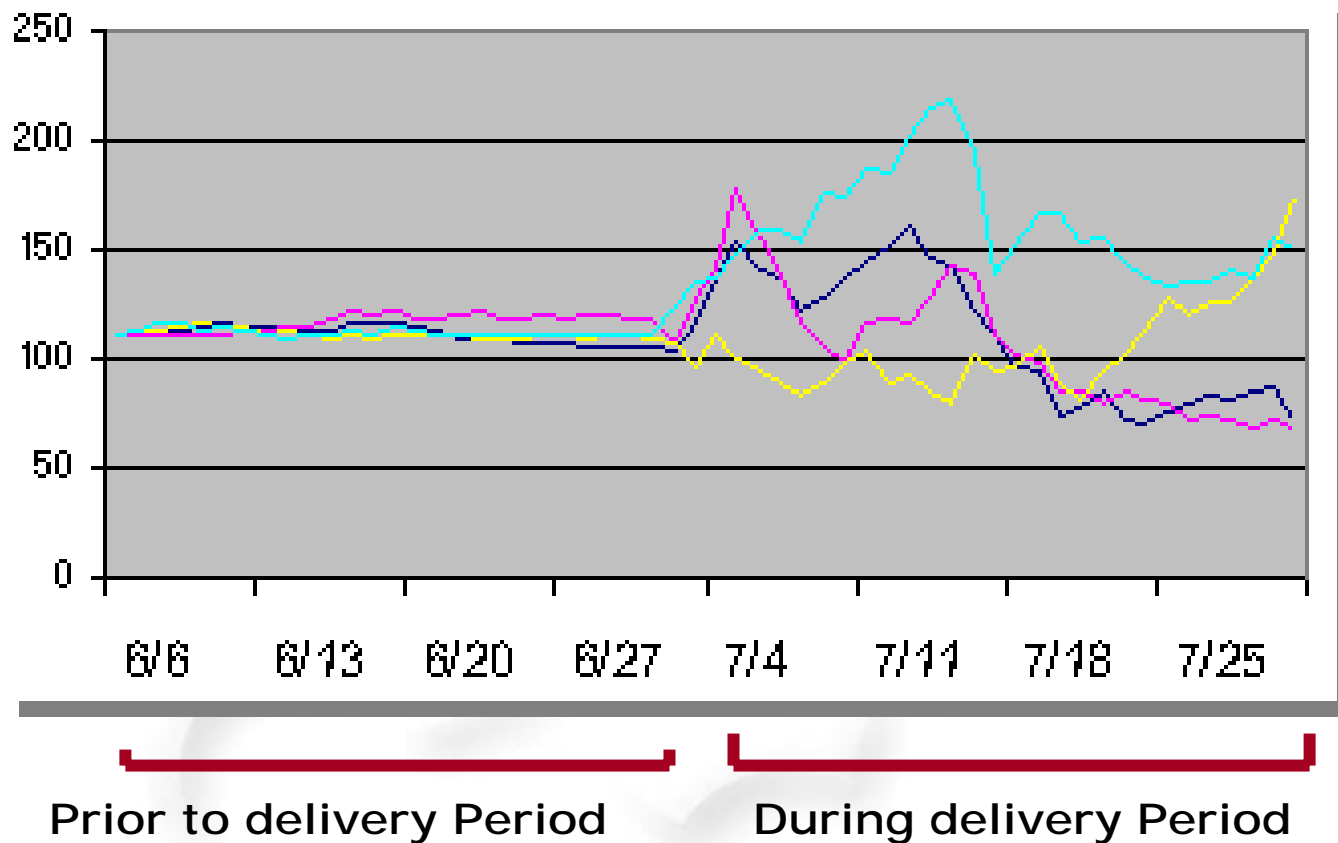


Time to Maturity Effect (Volatility)

- Forward/futures prices tend to be less volatile than spot prices



Integrated Volatility Term Structure



- Simulated Price Behavior.

Choice of Stochastic Price Process

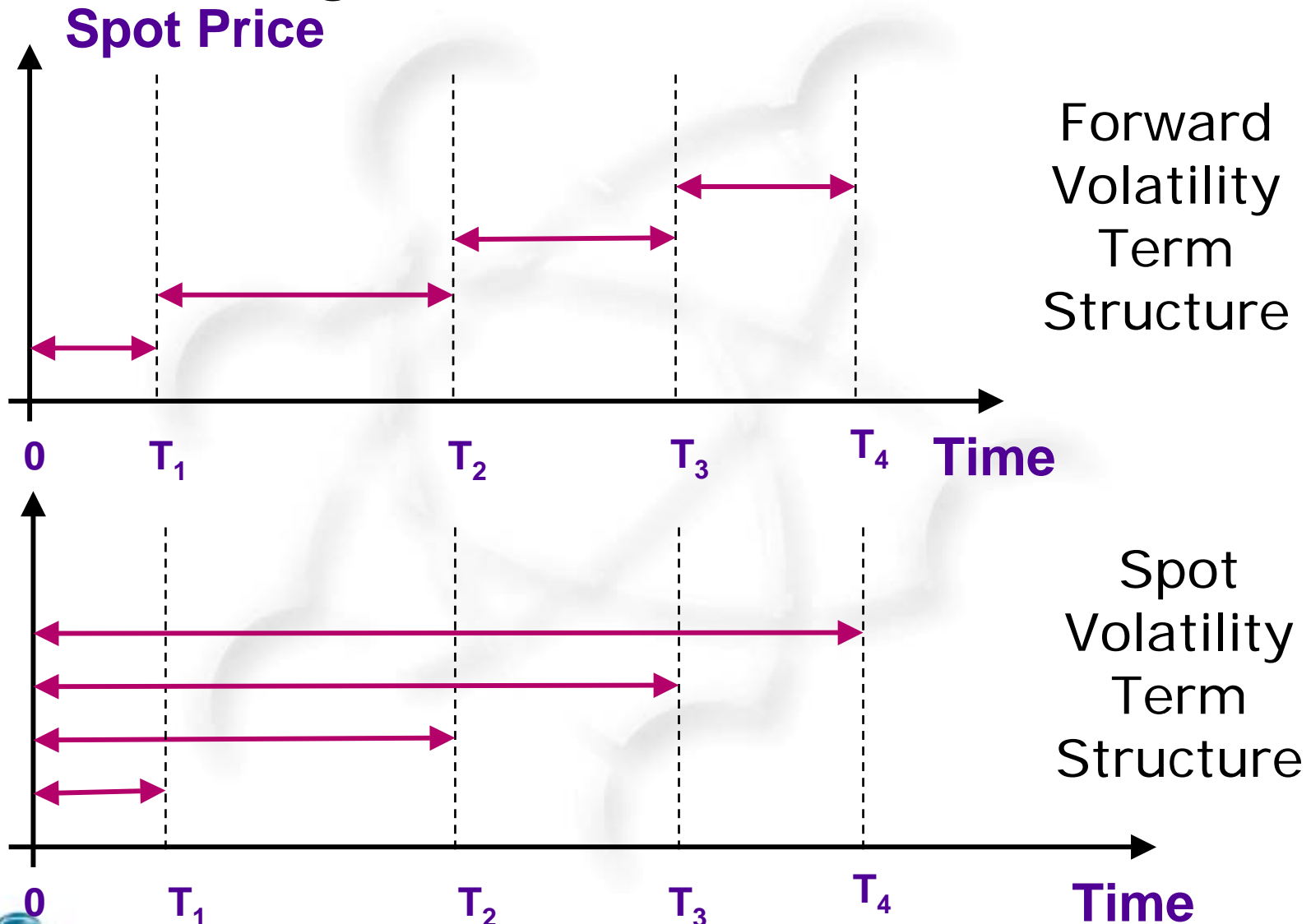
- Lognormal (Black-Scholes)
 - ✓ ➤ random movements, price bounded above zero
- Lognormal Mean-reverting
 - ✓ ➤ mean-reversion, seasonality, volatility backwardation
- Lognormal Mean-reverting with jumps
 - ✓ ➤ power spike behavior

Mean Reversion Price Dynamics

$$d \log S_t = a \left[\theta_t - \log \left(\frac{S_t}{F(0, t)} \right) \right] dt + \sigma_t dw_t$$

- S_t is the “spot” price of the commodity at time t
- w_t is a standard Brownian motion
- $a \geq 0$ is a constant mean-reversion rate (speed)
- σ_t is a time dependent local volatility
- $F(0, t)$ is the forward price at time zero for delivery of the commodity at time t .
- θ_t is a time -dependent function determined by requiring that the expected value of S at time t (determined at time zero), match the input forward price $F(0, t)$.

Volatility Term Structures



One-Factor Models

- Black-Scholes

$$\sigma_T(t) = \sigma(t)$$

- Lognormal Mean-Reversion

$$\sigma_T(t) = \sigma(t) e^{-a(T-t)}$$

if a long-term volatility is introduced additionally:

$$\sigma_T(t) = \sigma(t) e^{-a(T-t)} + \bar{\sigma} \left\{ 1 - e^{-a(T-t)} \right\}$$

Forward vs. Effective Volatility – No Mean Reversion

Forward Price Curve

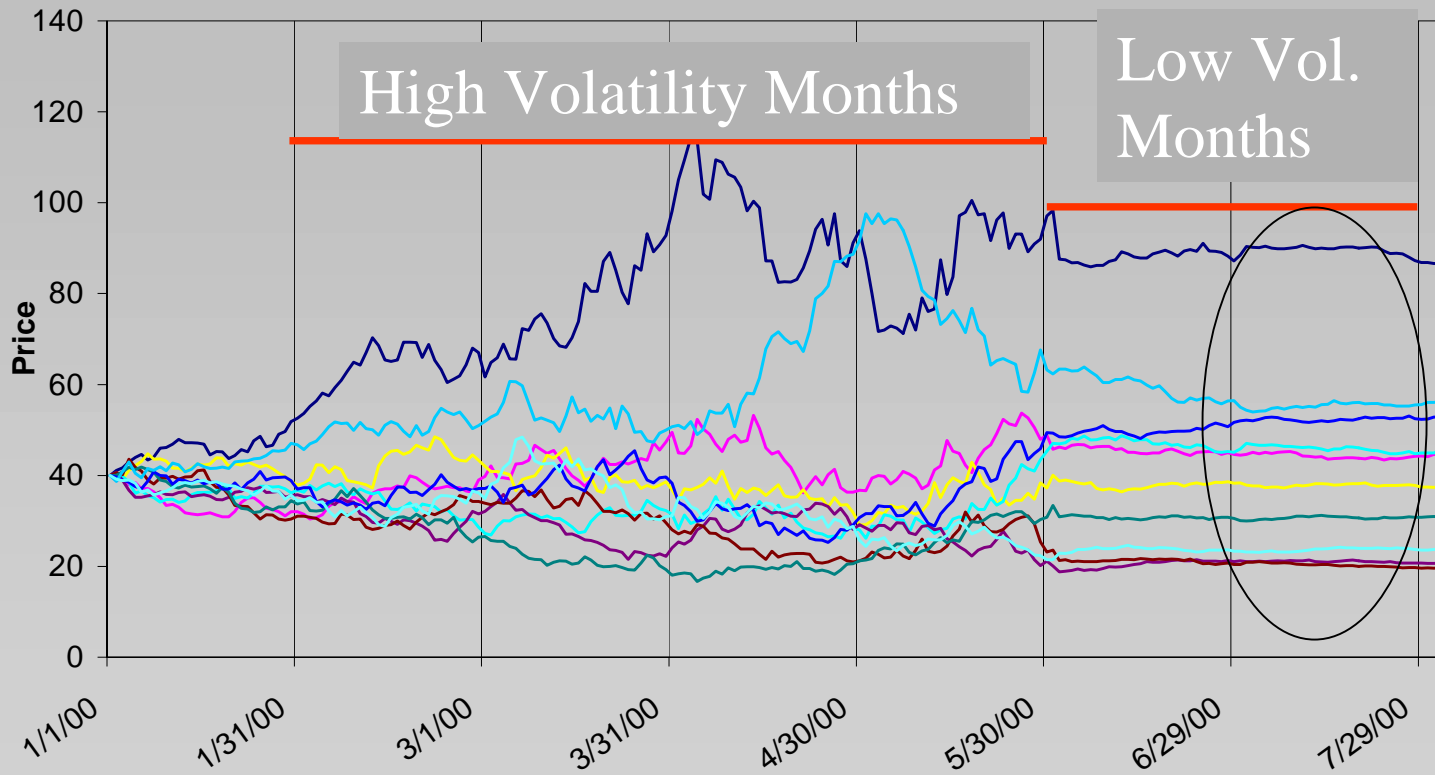
1/1/00	2/1/00	3/1/00	4/1/00	5/1/00	6/1/00	7/1/00	8/1/00
40	40	40	40	40	40	40	40

Volatility Curve

1/1/00	2/1/00	3/1/00	4/1/00	5/1/00	6/1/00	7/1/00	8/1/00
50%	50%	70%	80%	90%	100%	20%	10%

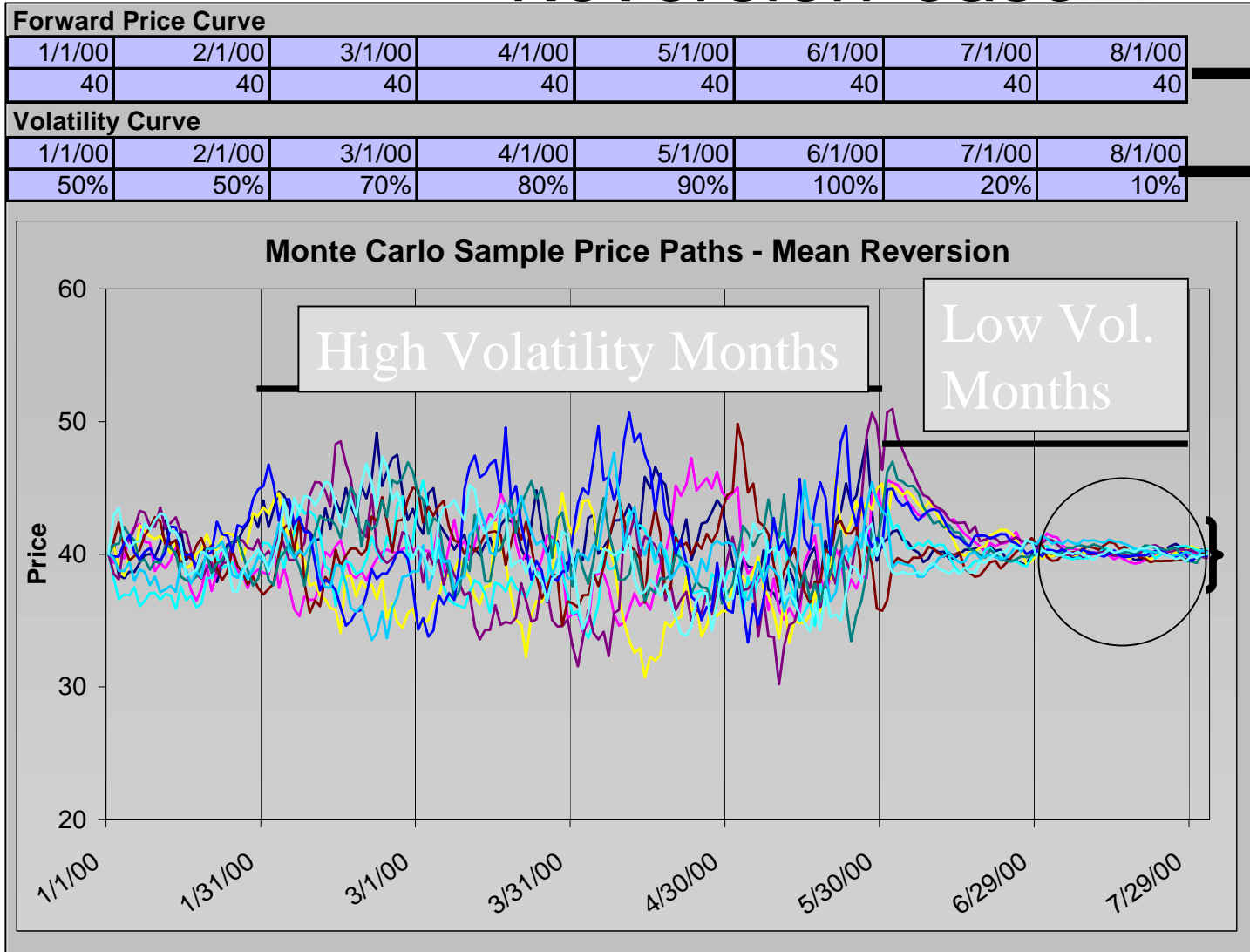
Forward Prices
Forward Vols.

Monte Carlo Sample Price Paths - Black-Scholes



In B-S models, price variability at a particular maturity (effective volatility) is heavily influenced by volatility in previous months

Forward vs. Effective Volatility - Mean Reversion Case



Forward Prices

Forward Vols.

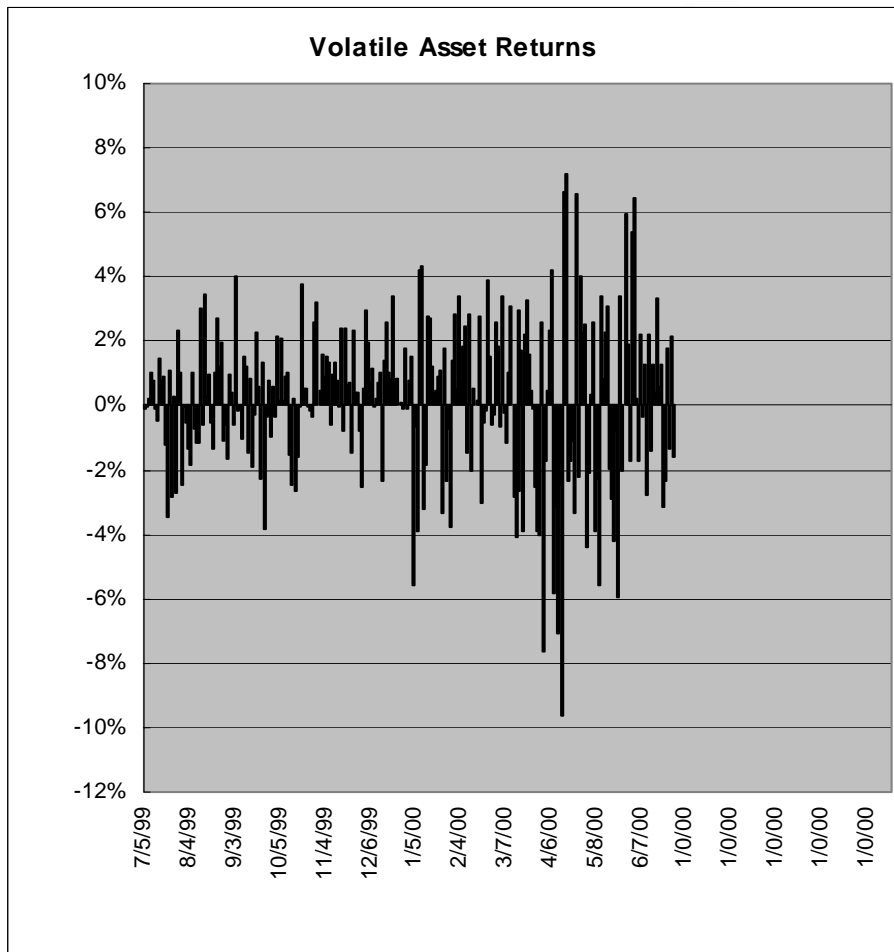
In Mean Reverting models, price variability at a particular maturity (Effective Volatility) can be independent of the spot volatility experienced in previous months

Jump-Diffusion Model

- All of the other attributes of the price process, but adds a jump-diffusion component
- Accounts for “price spikes” in electricity and other similar markets
- Utilizes three more parameters: (1) average size of jump, (2) frequency of jumps, and (3) variability of jumps.

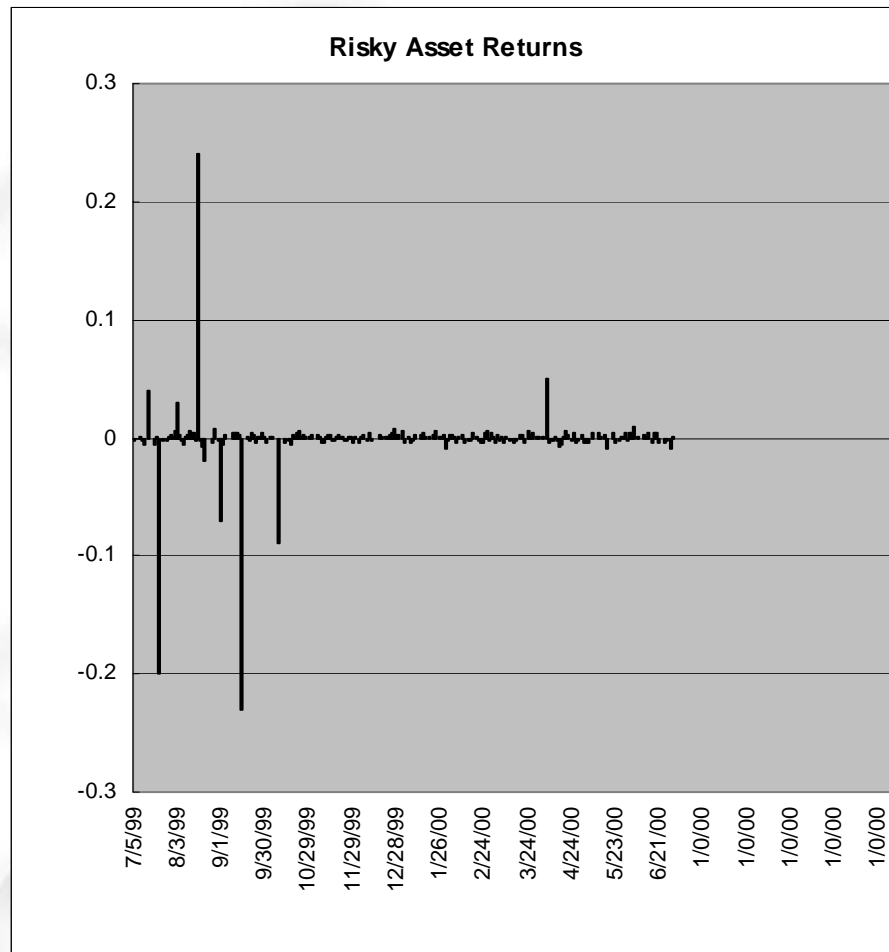
$$\dots + d\left[\sum_{i=1}^{N(t)} \tilde{X}_i\right]$$

Variability vs. Extreme Movements



Daily S.D.

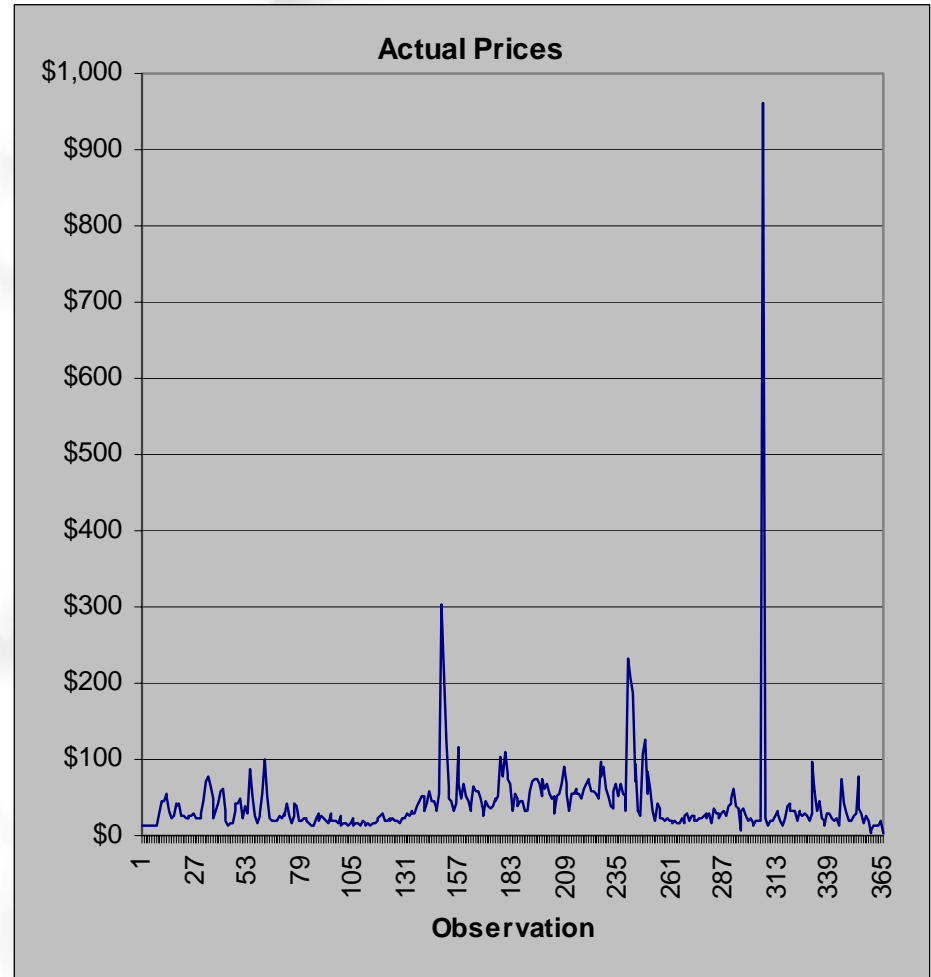
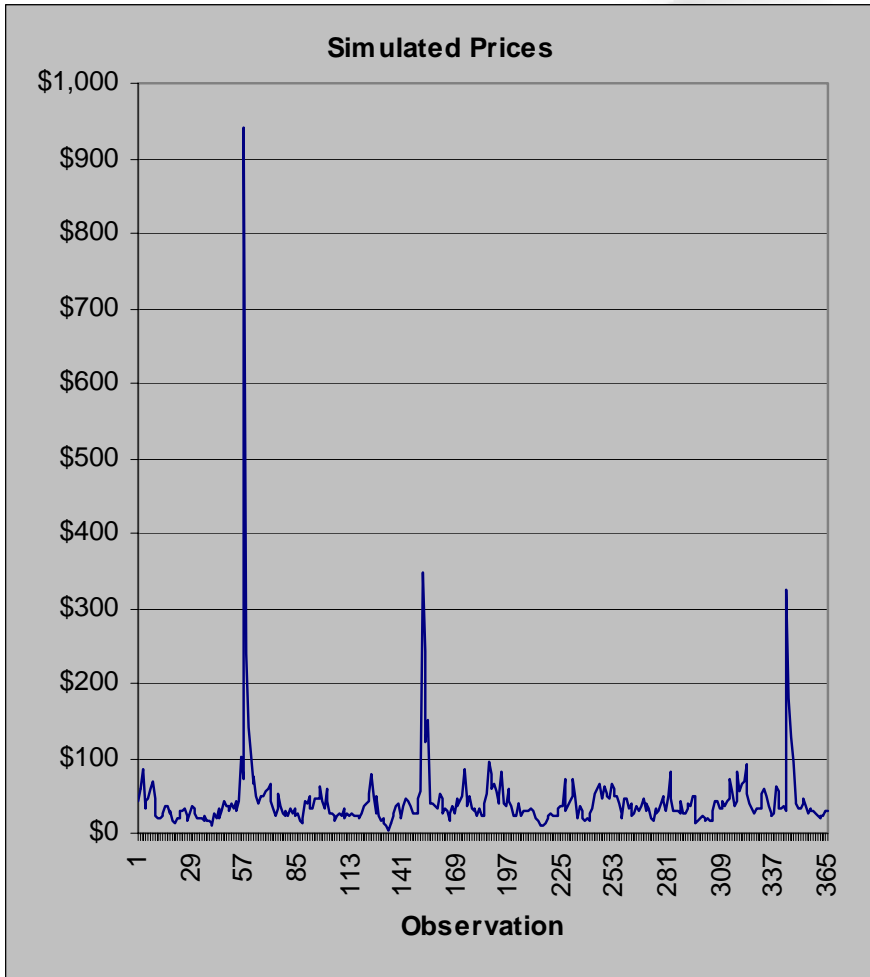
2.43%



Daily S.D.

2.57%

Actual vs. Simulated Prices for Victoria (Jump diffusion with mean reversion process)



Model Fit to Energy Price Behavior

Behavior	Black-Scholes	Mean Reversion	Jump Diffusion
Randomness	Yes	Yes	Yes
Mean Reversion	No	Yes	Yes
Mandated Price floors	No	No	No
Price Spikes	No	No	Yes
Fat tails	No	No	Yes
Seasonality in Prices and Volatility	Yes	Yes	Yes
Time to maturity effect	No	Yes	Yes