# Mean Reverting Processes – Energy Price Processes Used For Derivatives Pricing & Risk Management

This is the second article in a 3-part series exploring the main stochastic price processes used to model energy spot and forward prices for derivatives valuation and risk management. The first article in this series focused on the "random walk" assumption characterised by the most popular of price processes, Geometric Brownian motion, and the Black-Scholes option pricing model that is based on it. In this second article we shift our attention to the Mean Reverting Process, which incorporates the tendency of energy prices to gravitate towards a "normal" equilibrium price level that is usually governed by the cost of production and level of demand. By **CARLOS BLANCO** & **DAVID SORONOW**.

**WHY DO** we need to incorporate mean reversion when modelling energy prices? Suppose we observe that electricity prices jump from $30/MWh to $150/MWh due to an unexpected event (e.g. plant outages, transmission constraints, heat wave, etc.). Most market practitioners would agree that it is highly probable that prices will eventually return to their average level once the cause of the jump goes away. For similar reasons if the price of a barrel of WTI falls to US$7 due to overproduction we would expect the price to eventually rise as producers decrease supply. These expectations are intuitive in nature and are supported by our observations of energy spot price behaviour.

These two simple examples illustrate the limitations of Geometric Brownian motion (GBM) when applied to energy prices. In the electricity example above GBM would accept the US$150/MWh price as a normal event and would proceed randomly from there (via a continuous diffusion process) with no consideration of prior price levels (no memory), and no greater probability of returning to the average price level. This result is clearly at odds with market reality, and provides the impetus for the modelling of more complex mean reverting price processes.

## A Random Walk With Mean Reversion

Strictly speaking, the 'random walk' used to model prices under GBM is based on the assumption that price changes are independent of one another. In other words, the historical path the price followed to achieve its current price is irrelevant for predicting the future price path (prices follow a Markov process). Mean reversion can be thought of as a modification of the random walk, where price changes are not completely independent of one another but rather are related.

An analogy is often made that the random walk process is like a stumbling drunk after leaving the bar; the direction and size of his stumble are unknown and do not depend on his previous stride. Mean reversion can also be explained with a similar comparison. Now imagine that the drunk is being guided home by his German Shepherd dog, "Rusty". We want to determine the evolution of the distance between the drunk and the dog. The drunk will still stumble along in a random fashion. However, the size of his stumble is bounded by the length of the leash and the direction of his stride tends towards Rusty's position. When the drunk staggers away from Rusty, he will eventually be pulled back, and in due course will follow the path leading to his house.

## Random Walk With Mean Reversion: Mathematically

Mathematically, we capture the phenomena of mean reversion with a modification to the random walk assumption.

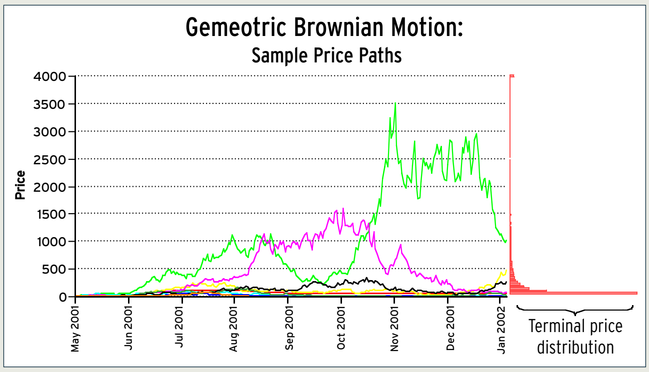
$$\underbrace{S_{t+1} - S_t}_{\boxed{\text{Expected change in price at } t+1 \text{ and } t}} = \alpha \underbrace{\left(S^* - S_t\right)}_{\boxed{\text{Mean Reversion Component}}} + \underbrace{\sigma \varepsilon_t}_{\boxed{\text{Random Component}}}$$

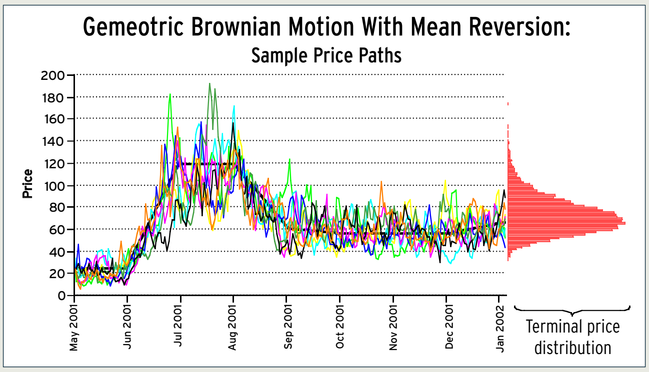
where:

$S^*$ is the mean reversion level or long run equilibrium price
$S_t$ is the spot price
$\alpha$ is the mean reversion rate
$\sigma$ is the volatility
$\varepsilon$ is the random shock to price from $t$ to $t+1$

Notice from this equation that the mean reversion component or 'drift' term is governed by the distance between the current price and the mean reversion level as well as by the mean reversion rate. If the spot price is below the mean reversion level, the mean reversion component will be positive, resulting in an upward influence on the spot price. Alternatively, if the spot price is above the mean reversion level, the mean reversion component will be negative, thus exerting a downward influence on the spot price. Over time, this results in a price path that drifts towards the mean reversion level, at a speed determined by the mean reversion rate.

The following graphs illustrate several simulated price paths assuming GBM vs. GBM with mean reversion. Notice that the GBM process produces prices that can reach unrealistic levels for long periods of time. Unlike pure GBM, the mean reverting GBM price process ensures prices gravitate over time toward the mean reversion price levels. Later in this article we will explore the implications of this property on derivative pricing and risk management.





Source: @Energy. Financial Engineering Associates

## Model Parameters: The Mean Reversion Level & Mean Reversion Rate

One of the great advantages of GBM models is the ease of estimating input parameters. The most difficult parameter to estimate is volatility: the expected future variability of price over time. This is typically estimated using either historical volatility, or implied volatilities from current option quotes. In contrast, mean reversion models require the estimation of additional unknown parameters such as the mean reversion level (the long-run equilibrium price), and the mean reversion rate (the speed at which prices revert).

Depending on the exact formulation of the Mean Reversion model, these parameters can often be extracted from current forward/futures prices and historical spot price series. A common approach is to assume the current forward/futures prices represent time dependent mean reversion price levels. The premise here is that forward/futures prices are the market's "best guess" (unbiased estimate) of future spot prices. This approach has the added advantage of incorporating seasonality in the price process since the term structure of forward prices usually has seasonal characteristics embedded within it.

There are several techniques for estimating the mean reversion rate. Linear regression can be used to relate historical price changes to historical prices. More recently, complex calibration techniques that fit model parameters to historical spot price data have been developed. Although it requires extra effort to determine these parameters, the mean reversion model provides a much more accurate depiction of market reality, especially for options pricing and risk management purposes.

The speed of mean reversion depends on several factors: the commodity being analysed, the delivery provisions associated with the commodity, and so on. In electricity markets, it is common to observe sudden price spikes with very fast mean reversion to the previous price levels before the price jump. In Natural Gas markets, the mean reversion rate is considerably

## FEA's @Energy's Mean Reverting Model (Extracted from @Energy User's Guide)

In FEA's @Energy, the following equation governs the evolution of $S_t$ in the mean reverting model without jumps:

$$d \log S_t = a \left[ \theta_t - \log \left( \frac{S_t}{F(0,t)} \right) \right] dt + \sigma_t \, dw_t \qquad (1)$$

where $S_t$ is the 'spot' price of the commodity at time $t$, $w_t$ is a standard Brownian motion, $a \geq 0$ is a constant mean reversion rate (speed), $\sigma_t$ is a (possibly) time-dependent local volatility, $F(0,t)$ is the forward price at time zero for delivery of the commodity at time $t$, and log denotes the natural log.

The time-dependent function $\theta_t$ is internally determined by requiring that the expected value of $S$ at time $t$ (determined at time zero), match the input forward price $F(0,t)$. This guarantees that the process used is consistent with market prices of forward and futures contracts, and is required for arbitrage-free pricing of the derivatives valued by @ENERGY.

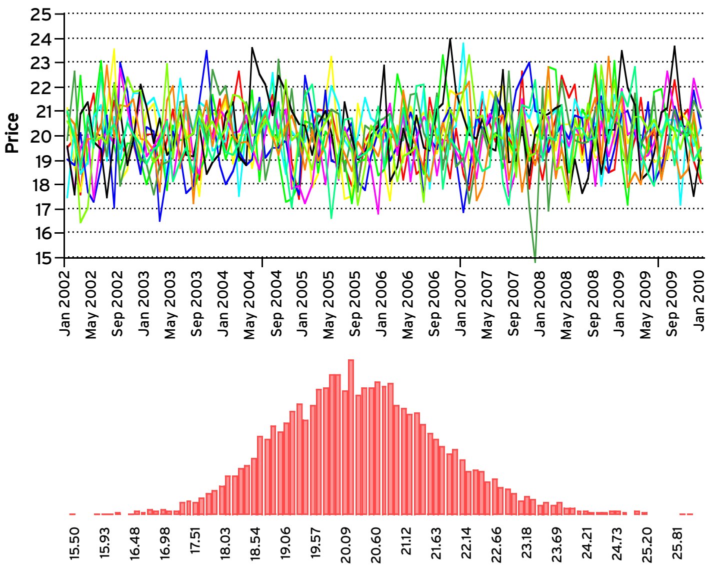
Equation (1) provides a way to compute the evolution of the full forward price curve by using information about:

- The initial forward price curve $F(0,t)$ for $t \geq 0$.
- A mean reversion rate.
- An estimate of the volatility of $S$ as a function of time (technically, an instantaneous forward volatility curve).

The mean reversion rate is always greater than, or equal to, zero and higher numbers correspond to faster mean reversion. The mean reversion parameter, $a$, represents the annualised rate at which the underlying short-term price returns to its expected value. Hence the inverse of $a$ gives the actual time scale over which mean reversion occurs. For example, a mean-reversion rate of 2 corresponds to a commodity whose price reverts to its expected value over the course of six months (i.e., 1 / 2 years).

### FEA – Random Price Simulator–Mean Reverting Lognormal

| Market Inputs | Value |
|---|---|
| Forward Price | 20 |
| Volatility | 44% |
| Spot Price | |
| Interpolation | 1 |
| Futures Contract | |
| **Simulation Inputs** | **Value** |
| Simulation sate date | 01/01/02 |
| Simulation end date | 01/01/10 |
| Sampling Frequency | m |
| Seed* | |
| Simul* | 10000 |
| Min-Max* | |
| **Model Inputs** | **Value** |
| Model | |
| Mean-Revision rate | 24 |
| Cap on Price | |
| Floor on Price | |
| Yearly jump frequency | 2 |
| Expected jump size | 0.6 |
| St. Dev. of jump | 0 |
| **Other Inputs** | **Value** |
| Value Date* | 01/01/00 |
| Interest rate | 0.00% |



slower, but the volatilities for longer-dated contracts are usually lower than the volatilities for the shorter-dated ones. In oil markets, the mean reversion rate is thought to be longer term, and it can take months, or even years, for prices to revert to their mean.

## Implications For Derivatives Valuation & Risk Management:

### *Option Pricing*

The mean reversion model has significant implications for option pricing, especially for American exercise options and certain exotic instruments such as barrier options and swing options.

It is common practice for traders to estimate volatility by calculating the Black-Scholes model implied volatility from a set of quoted European exercise option prices. These volatilities are then used to price less liquid exotic options such as barriers, spread options, and swing options using GBM based option pricing models. However, many practitioners may be unaware that this approach can produce erroneous results, especially in high price volatility environments such as is the case in most power markets.

Consider a natural gas call option that 'knocks-out' and becomes worthless if the price of natural gas rises above US$12/mmBtu at anytime during the life of the option. Using a GBM price process to model natural gas prices produces price paths that result in a much higher probability of reaching the barrier level during the option life than a mean reverting price process does. Option pricing models that use a mean reverting price process ensure prices gravitate towards the mean reversion level, thereby assigning less probability of touching the barrier during the life of the option.

The graphs below illustrate simulated price paths and resulting histograms for natural gas using GBM vs. GBM with mean reversion. The higher prices produced by the pure GBM method can be clearly seen. In this example, both price processes produce the same result for a European exercise option, but drastically different option prices for a barrier option.

## Mean Reversion & Real Options Analysis

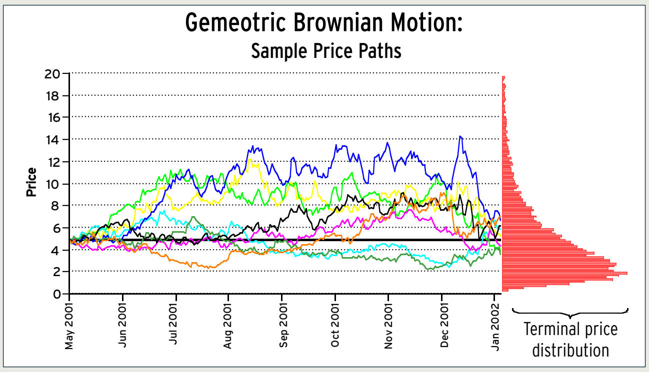
Another example of the dangers of not taking account of mean reversion comes from asset valuation. Many companies are starting to treat energy related fixed assets as derivative instruments using an approach known as "real options analysis". The downside of ignoring the derivatives pricing approach has been demonstrated dramatically in US power markets. Many utility companies who were attempting to purchase cogeneration plants found themselves outbid by competitors. In such cases, these competitors felt confident in bidding higher because their grasp of the derivatives viewpoint gave them a greater understanding of the value of the plant. The losing utilities had instead relied on traditional NPV analysis.

For asset valuation and risk analysis, we can follow a two step process. Firstly we decompose the Fixed Asset as a set of exposures to market factors. Our profit/loss each day is going to be a function of the spot prices recorded that day, the power generated, load, etc. In the second step we simulate the different variables that determine our hypothetical profit/loss for that day.

For example, suppose that a generation facility produces electricity by burning natural gas. From a derivatives perspective, the operator has the option to transform gas into electricity. In derivatives vernacular, the generation plant is nothing more than a basket or "strip" of spread options, since such choices can be repeatedly made during the facility's useful life. From a risk management point of view, the operator has exposures to power and natural gas prices. He will be short natural gas (because he needs natural gas to fire his generators), and he will be long power (because he has the ability to produce electricity). But his exposure to electricity, as shown by the power and natural gas option deltas, wouldn't necessarily be 100% of the output of the plant since he can decide not to run the plant.

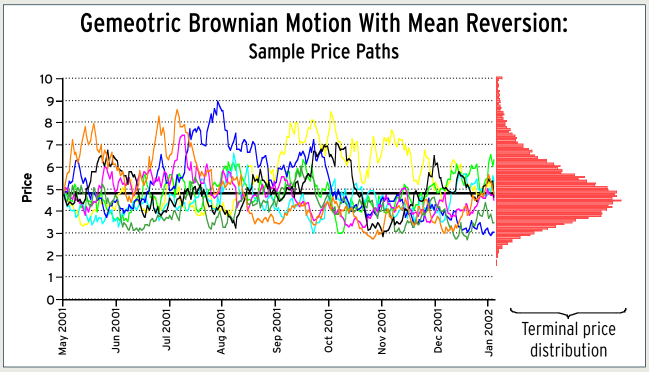
In order to value this "derivative" it is essential that our possible price scenarios are plausible. Unless we use the right stochastic process, our simulated profits could be totally unrealistic. If we used a "random walk" process to simulate electricity prices for the next 10 years, we would find that the simulated prices after the first few months would not accurately represent their expected behaviour.

Another important aspect is the "greek letters" or sensitivities of the derivative to the various components determining its value. If we wrongly assume that prices follow a non-mean reverting process, we would find that the greek measures become meaningless in a hedging context, especially for assets with very high volatilities. For example, if we use GBM to value European options with very high volatilities, the delta converges to 1 for call options and –1 for put options. This is also the case for the deltas in Spread Options where one of the assets has very high volatility. The deltas of the underlying assets (e.g. power and gas) converge to 1 and –1 respectively, due to the erroneous choice of the right price process to determine those price sensitivities.



Gemeotric Brownian Motion:

Sample Price Paths



Gemeotric Brownian Motion With Mean Reversion:

Sample Price Paths

Source: @Energy. Financial Engineering Associates

If we use a mean reverting process, we would obtain more accurate "deltas" or price sensitivities that would produce a more effective hedge strategy.

### Value at Risk & The "Square Root Of Time Rule"

Our final example of the problems with GBM concerns the square root of time rule commonly used in Risk Management to convert certain Value at Risk measures between different holding periods. If prices follow a mean reverting process, the "square root of time rule" would give us very large volatility estimates. Note that in a mean reverting process, the volatility has a limited effect on prices, and due to the fact that after a shock, prices revert to their long run levels, their long run variability does not grow proportionately with time.

### How To calculate Mean Reversion Rates, Mean Reversion Levels & Volatilities In Excel

This practical example illustrates the problem with the square root of time rule.

1. Choose a particular price series. Prices could be hourly, daily, weekly, etc. Column A (rows 2–11) contains historical prices for asset x over a ten-day period.
2. Calculate the standard deviation assuming that returns are independent. Column B (rows 3–11) shows the daily returns, denoted u. Use Excel's built-in STDEV (sample standard deviation) function to calculate the daily volatility (cell B13). Using the square-root-of-time rule, we annualise to obtain s* (cell B15) assuming that time is measured in trading days and there are 250 trading days per year. In our example, the annualised volatility would be equal to 265%, (16.7% x √250 ).
3. Calculate the absolute price changes. Column D (rows 3–11) shows the daily changes.
4. We can estimate the Mean Reversion rate in a relatively simple and robust manner by regressing absolute price changes (Column D) on the previous price levels (Column E).
5. Use the Excel functions SLOPE, INTERCEPT and STEYX (residual standard deviation) to calculate the parameters from the regression. The mean reversion speed is the negative of the slope, while the long run mean is the intercept

**Mean Reversion Rates, Levels & Volatility Calculations**

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | Date | Current Price, $ | Price Change, % | Price Change, $ | Previous Price, $ |
| 2 | 5/1/01 | 15 | | "y" Values | "x" Values |
| 3 | 5/2/01 | 18 | 18.2% | 3 | 15 |
| 4 | 5/3/01 | 15.5 | -15.0% | -2.5 | 18 |
| 5 | 5/4/01 | 12 | -25.6% | -3.5 | 15.5 |
| 6 | 5/5/01 | 14.5 | 18.9% | 2.5 | 12 |
| 7 | 5/6/01 | 13 | -10.9% | -1.5 | 14.5 |
| 8 | 5/7/01 | 15 | 14.3% | 2 | 13 |
| 9 | 5/8/01 | 17 | 12.5% | 2 | 15 |
| 10 | 5/9/01 | 15.5 | -9.2% | -1.5 | 17 |
| 11 | 5/10/01 | 14 | -10.2% | -1.5 | 15.5 |
| 12 | | **Standard Deviation** | | **Regression Parameters** | |
| 13 | | STDEV(u) | 16.7% | SLOPE | -89% |
| 14 | | SQRT(250) | 15.87 | INTERCEPT | 13.2 |
| 15 | | S* Annualised | 265% | STEYX | 1.98 |
| 16 | | | | | |
| 17 | | Long Run Mean (Intercept/Speed) | | | 14.9 |
| 18 | | Volatility (STEYX/Long Run Mean) | | | 13.2% |

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estimate of that regression divided by the mean reversion speed.

6. The volatility of dollar price changes is given by the residual standard deviation calculated with STYDX. If we want to obtain a percentage volatility, we would just need to divide by the long run mean (see E18).

In contrast with the "random walk" process presented in the previous issue, where price changes were independent through time, a mean reverting process is characterised by prices that have some degree of memory about previous price changes. The mean price levels will be our best forecast of future price levels. In the above example, the mean reversion level is 14.9 (Intercept value divided by the speed).

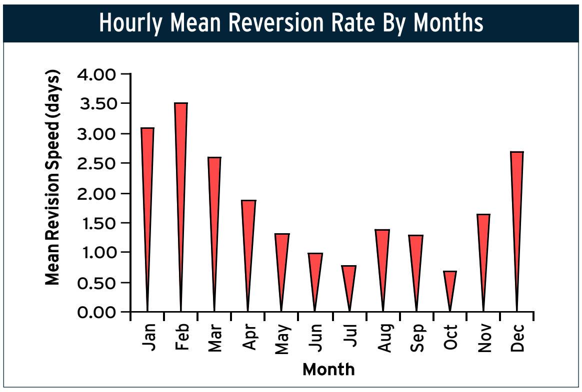
The volatility of our forecast would be around 13.2% of the forecast price level, which is substantially lower than the 265% calculated with the square root of time rule.

## Pitfalls Of Using Mean Reversion Processes To Model Mean Reversion:

### *1. Energy Prices exhibit price jumps not described by the log normal distribution*

Energy prices in general and electricity prices in particular, diverge significantly from the log normal distribution. Although Geometric Brownian motion with mean reversion adequately models the way in which prices diffuse back towards the long-run equilibrium level after a jump event (eg. unexpected outage, extreme weather, etc.), it fails to capture the jump event itself.

This pitfall becomes extremely important for risk management and pricing of exotic and deep out-of-the-money options, and the valuation of assets such as certain peaking plants that are only turned on in extreme price scenarios.


Hourly Mean Reversion Rate By Months

### *2. Mean reversion rates are not constant*

The speed at which prices revert to their long run levels may depend on several factors such as the nature, magnitude and direction of the price shock. If we calibrate the mean reversion rate for each month of the year using data exclusively from that month, we would find that for most markets the mean reversion rates differ considerably.

In the following chart, we can see the mean reversion speed for hourly prices calculated for each month of the year using Nordpool hourly price data and FEA's Forward Curve Builder.

We can clearly see how the mean reversion rate is different for each month. When using the model for valuation or risk management purposes we need to make a choice between using a mean reversion rate specific to each month or an average one extracted from the whole data sample.

## Conclusion

Mean reversion in energy prices is well supported by empirical studies of energy price behaviour, as well as by basic microeconomic theory. General diffusion models that incorporate mean reversion go a long way in capturing the nature of energy prices; notably their tendency to randomly oscillate away from, and over time back towards a price level determined by the cost of production. These models are gaining more widespread acceptance among market practitioners as advances are made in the techniques used to estimate the mean reversion level and mean reversion rates. The advantages of mean reversion option pricing models over their Black-Scholes counterparts are becoming more and more apparent as traders and risk managers are able to assign greater accuracy to their models, or at a minimum to their model assumptions.

Unfortunately, these models fail to describe what is, for risk management purposes, perhaps the most important energy spot price phenomenon: the discontinuous price jump. In the next article, we will explore the nature of these jumps and the jump diffusion price process that describes this behaviour ▪

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