LEARNING CURVE *EXTREME VALUE VaR (2)*[*]

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Traditional approaches to VaR estimation are inadequate because they do not take proper account of the more extreme observations in our data set. This shortcoming can be rectified by using Extreme Value (EV) theory, which provides a tailor-made approach to the estimation of low frequency events with limited data, and the resulting EV approach to VaR was outlined in last week's *Learning Curve*.

Applying EV to Portfolios

However, the existing literature on EV-VaR has tended to focus on applications to a particular risk factor (e.g., to an individual equity index, commodity price, etc.). This is a serious limitation because risk managers are interested, not so much in different risk factors taken in isolation, but in how risk factors interact to affect the risk of the portfolio as a whole. If we are to apply EV-VaR to portfolios, we must therefore take these interactions into account. Unfortunately, this is easier said than done, because risk factors can interact in complex ways, particularly under extreme market conditions.

One solution to this problem is use average correlations to capture the interactions. However, the relation between risk factor changes and correlations is often asymmetric and non linear, and the use of average correlations will miss these features and so produce unreliable risk measures. To some extent, we can deal with this problem by using modeling procedure that deal with asymmetry and non-linearity (e.g., such as E-GARCH). But such approaches can only take us so far, because they don't take account of the correlation structure of extreme events, and it is a well-known fact that correlations can change drastically during market crises – extreme correlations and average correlations can be very different

A natural solution is to make use of multivariate extreme-value theory (M-EVT), which is concerned with modeling the tails of multivariate distributions in a theoretically supported fashion. However, M-EVT models are only viable in a small number of dimensions. Imagine that we had 1000 observations of two variables x_t and y_t, and we were interested in the most extreme 1% of observations. We would therefore have 10 extreme observations for each variable. But how often would x_t take an extreme value at the same time as y_t? Not often, clearly. If extreme values are rare, matching extremes are rarer still. Moreover, the greater the number of variables, the rarer the matching extremes become. This 'curse of dimensionality' makes it very difficult to estimate extreme correlations accurately, because we rapidly run out of the matching observations we would need to do so.

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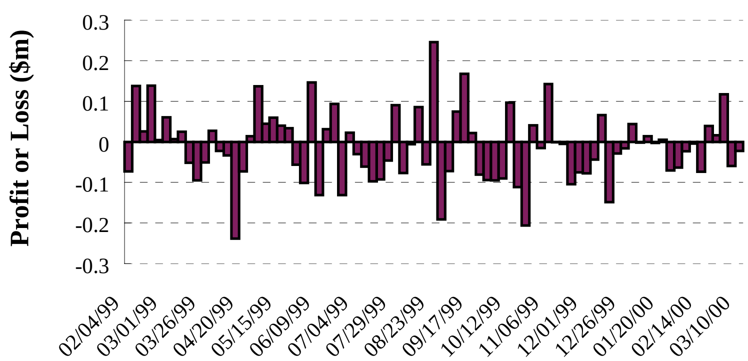
A better approach is to mix the historical simulation (HS) and single-factor EV approaches to VaR. The beauty of the HS approach is that it is not based on inappropriate correlation assumptions, and so preserves the correlation structure of extreme events in our data. However, the traditional HS approach does not allow us to project out beyond our sample range. This is where the EV approach comes in.
In effect, we use the HS histogram to estimate the parameters of our EV distribution, and then use the EV theory to project the tail out beyond our sample, thereby allowing us to estimate extreme VaRs and the probabilities associated with them.

An Illustration of the HS-EV Approach to Portfolios

For example, suppose we have the data in Figure 1. These data represent the daily profits or losses on a typical commodity swaps desk over a period of just over a year.

Figure 1: Daily P&L



To apply the HS-EV approach, we begin by choosing a VaR confidence level, which determines the size of our tail. We then choose a suitable EV distribution, estimate the parameters and plug these into the relevant formula to derive our VaR estimate. For example, if we choose a 95% confidence level and use a Generalized Pareto EV distribution, we will get a VaR of about $13.5m.

The same approach can also be used to estimate the Expected Tail Loss (ETL) associated with our VaR estimate – the loss we would expect in a tail event (i.e., in the worst 5% of outcomes). The ETL is a natural complement to VaR and is, in some respects, a better risk measure. In this case, the ETL is $17.8m, or $4.3m more than the VaR.

Main Points

The new approach suggested here has two big attractions – it is theoretically correct, in the sense that it makes use of EVT in a theoretically sound way, and it is easy relatively easy to implement.

However, to make good use of this approach, we must also bear in mind that we need a decent run of data, typically several years or more of suitable daily historical observations. This condition is easily met in many markets, but can be a problem in immature markets (e.g., energy, IPOs, etc), or markets that experience structural changes (e.g., changes in exchange rate regimes).

We should also bear in mind that standard EVT analysis only provides a conditional estimate of our EV-VaR, and this estimate is conditional, among other things, on our estimates of the extreme correlations. These are very difficult to estimate precisely, as we have seen; moreover, changes in market behaviour (e.g., hedging strategies) can also lead to changes in extreme correlations and therefore major errors in our estimates of EV-VaR.

Finally, all HS-based approaches to VaR – ours included – suffer from the limitation that they only pay attention to real historical events. They pay no attention to events that could plausibly have occurred, but did not actually occur, in our data set. For this reason, it is important to supplement HS approaches – and our HS-EVT approach too – with sound stress tests to determine what we would stand to lose under various 'what if' scenarios.