

CHARM SCHOOL

Charm is just an ad hoc measure of risk - isn't it?

Mark Garman thinks not

Traders commonly use the term "carry" to describe the expense of financing the deferred delivery of currencies, commodities, and other assets in financial contracts. For example, consider a forward contract to purchase gold for delivery at a later date, paying dollars at another date. Since the contract involves two deliveries that occur unconditionally, it is relatively easy to assess the carry, or cost of financing each leg of the transaction, by estimating the cost of transforming current gold into gold at its delivery date, and next the current dollars into their equivalent at delivery date, then reducing these costs to current accounting units. Carry is supposed to pertain to each individual asset (leg), providing a decomposition of costs.

But suppose that the asset deliveries are contingent, as in an option contract; or that the contingency is tied in a complicated fashion into perhaps three assets, as in a rainbow option to exchange dollars for one's choice of gold or silver. What then is the cost of carry for each leg in such a complex financial contract?

Charm is a new risk measure that rigorously defines the concept of carry in the most complex financial instruments, including even exotic options. It preserves the notion of dividing a financial instrument's time value changes by individual asset involvement. However, charm was discovered and named in an entirely different context.

Charm arose as a name for an ad hoc risk measure being used by derivative traders. In fact, three such measures were casually observed and subsequently named as follows:

Speed: $\partial\Gamma_i / \partial X_i$

Charm: $\partial\Delta_i / \partial t$

Colour: $\partial\Gamma / \partial t$

Speed answers the trader's question: "How much will my position's gamma change if a price change occurs?" This is a reasonable question for anyone who trades derivative products, but since speed is a third-order term there is strong evidence

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that it has little or no theoretical role in modern finance (at least in no-arbitrage, continuous-time models).

Charm answers the question: "How will my delta change if time passes and prices remain the same?" This may be important to a trader. If, for example, he is engaged in delta-neutral hedging, he may have to trade tomorrow to correct a changed delta, even though nothing else happens in the market. Again, since no such term as charm appears in the basic equations of continuous-time finance, it was my initial perception that it was merely ad hoc, and could play no possible theoretical role.

This turns out to be incorrect.

Colour answers the question: "How will my gamma change if time passes and prices remain the same?" Again, no theoretical role would normally be forecast for this risk measure.

Focusing on charm, consider the payoff diagram for one of the most common derivative products, a European call option (figure 1). In this figure,¹ $A(X,t) = \partial C(X,t) / \partial X$ (delta), being the slope of the call option valuation function with respect to the asset price X , will change as time passes. If X is less than the option strike price K , the slope will tend to decline as time passes.

¹Note that the valuation function is drawn so that it penetrates the conversion value (heavier line), which means that the underlying asset X has some positive payout or interest rate, an important component of its carry

²The strike price K is not necessarily the exact changeover point

³See R Merton, *Theory of rational option pricing*, Bell Journal of Economics and Management Science, 4 (1), 1973, pages 573-615

⁴In some exotic options, such as in-progress average price options, the accounting asset enters surreptitiously, and must be accorded its own charm component

Conversely, if X is larger than K , the slope will tend to increase as time passes.² In other words, here we see that charm tends to be negative for out-of-the-money call options, and positive for in-the-money call options, as illustrated in figure 2.

The surprise is that charm does not merely provide a few traders with an ad hoc measure of how their delta may change overnight, but also formalises the notion of carry by dividing theta into its asset-based constituents.

A basic theoretical result regarding charm is that: *dollar charm (ie, charm multiplied by price), taken over all the assets that affect the value of a derivative, is conserved by theta.*

This theorem implies that dollar charm is the natural division of theta into its asset components, and hence is the natural definition of carry. Expressed somewhat more formally, the theorem says that:

$$\sum_{i=1}^N X_i h_i = \Theta$$

where $h_i = \partial \Delta_i / \partial t$

That is, h_i is the charm associated with asset i and X_i is the price of asset i .

The proof of the theorem is surprisingly easy, and relies upon the value conservation equation³ (also known as the replication equation), which states that

$$\sum_{i=1}^N X_i \Delta_i = V$$

(This equation says that by pursuing a strategy in which he replicates a security that depends only upon the asset prices X_i by buying - or selling - the amount dictated by the corresponding delta, the trader will have made the same investment regard-

less of whether he replicates in this fashion or purchases the security outright.) All that is required to flesh out the proof is to take the time derivative of both sides of the value conservation equation.

This theorem is widely applicable. It works for all of the orthodox derivatives, such as put and call options, and also for exotic derivatives like lookbacks, knock-outs and rainbows.⁴

As an aside, it may be pointed out that the prominence of the more usual Greek-letter risk measures (delta, gamma, theta) is due to their presence in equations of motion (ie, generalised Black-Scholes equations) for derivative products:

$$\frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N \sigma_i \sigma_j \rho_{ij} X_i X_j \Gamma_{ij} + \sum_{i=1}^N (r - r_i) X_i \Delta_i + \Theta = rV$$

where

V = value of contract

X_i = price of asset i

σ_i = instantaneous standard deviation of X_i

ρ_{ij} = instantaneous correlation coefficient between X_i and X_j

Γ_{ij} = gammas (when i equals j) or cross-gammas (when i does not equal j)

r = instantaneous rate of return on accounting commodity

r_i = instantaneous rate of return on asset i

Δ_i = deltas

Θ = theta

However, we can substitute into this the charm conservation theorem above to yield a new, and potentially more separable equation of motion as follows:

$$\frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N \sigma_i \sigma_j \rho_{ij} X_i X_j \Gamma_{ij} + \sum_{i=1}^N X_i \{h_i - r_i \Delta_i\} = 0$$

This equation shows that derivative product hedging is really linked to assets other than the underlying ones only through correlation coefficients, and so provides a basis for cross-hedging - for example, the hedging of a call option on Dutch guilders using Deutschmarks.

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RISK

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